Centrum Wiskunde & Informatica

Image Reconstruction - A Playground for Applied Mathematicians

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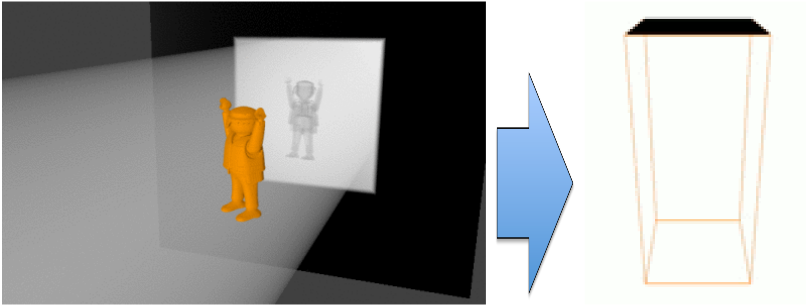
Applied Analysis Seminar
Radboud University
24 Feb 2022

Introduction and Overview

Computational Imaging @ CWI

- headed by **Tristan van Leeuwen**, 18 members
- mathematics, computer science, (medical) physics & engineering
- advanced computational techniques for 3D imaging
- (inter-)national collaborations from science, industry & medicine
- one of the two main developers of the **ASTRA Toolbox**
- **FleX-ray Lab**: custom-made, fully-automated **X-ray CT** scanner linked to large-scale computing hardware

X-ray Computed Tomography (CT)



- X-rays (high-energy photons) get **attenuated** by matter
- 3D attenuation image **computed** from different 2D projections

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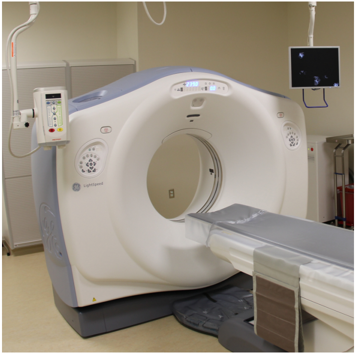
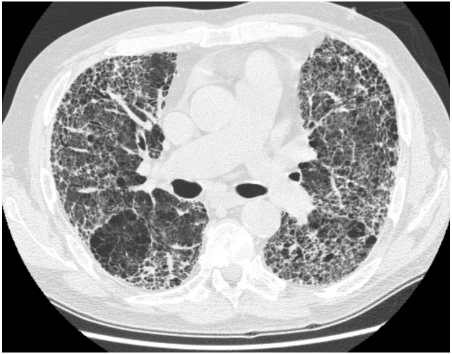


(a) Modern CT scanner



(b) CT scan of a patient's lung

Source: Wikimedia Commons

Imaging Across Disciplines

Observational astronomy

Life and material science

microscopy

Medical imaging

CT, MRI, US, PET, SPECT...

Geophysical imaging

(electrical) resistivity, seismic

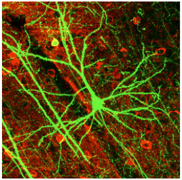
(ground-penetrating) radar...

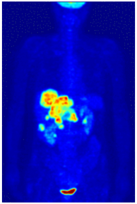
Remote sensing

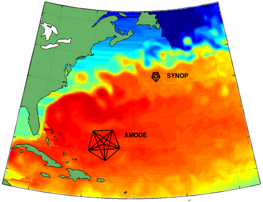
military/intelligence,

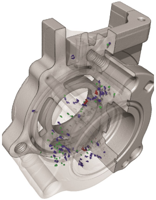
earth/climate science

Industrial process imaging









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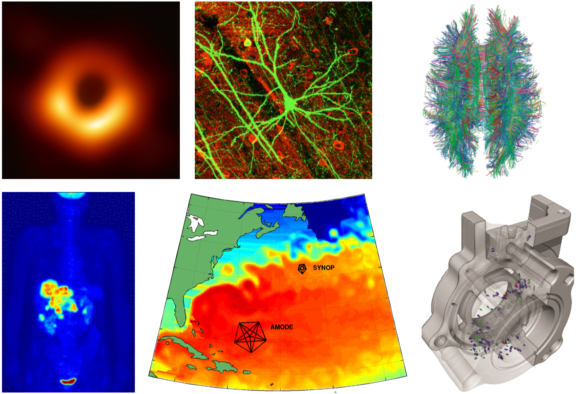


Source: Wikimedia Commons

Mathematical Imaging: *Reconstruct spatially distributed of quantities of interest from indirect observations through algorithms derived from rigorous mathematics.*

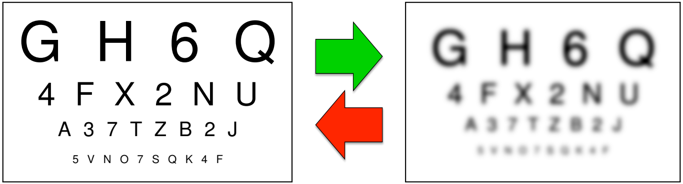
Imaging: An Inverse Problem

Inverse problem: Recover **unknowns** u (image) from **data** f via

$$\boxed{f = A(u) + \varepsilon}$$

- **Forward operator** A solution of **PDE** modelling underlying physics.

Imaging: An Inverse Problem



u $\qquad$ $A(u)$

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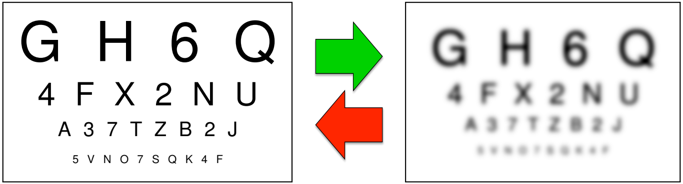

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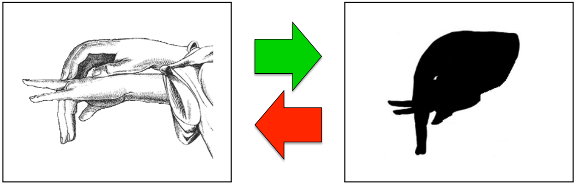

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Overview Inverse Problems / Imaging Workflow

mathematical modeling:
physics, PDEs, approximations

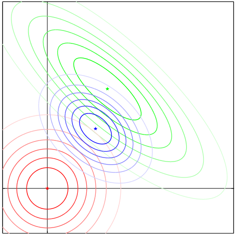
reconstruction/inference approach:
regularization, statistical inference, machine learning

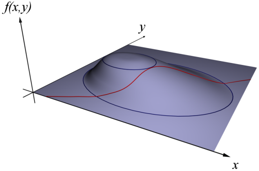
theoretical analysis:
uniqueness, recovery conditions, stability

reconstruction algorithm:
PDEs, numerical linear algebra, optimization, MCMC

large-scale computing:
parallel computing, GPU computing

$$(s \cdot \nabla + \mu_a(x) + \mu_s(x))\, \phi(x, s) = q(x, s) + \mu_s(x) \int \Theta(s, s')\phi(x, s')ds'$$





Current Challenges in Computational Imaging

core development for new modalities:
hybrid imaging

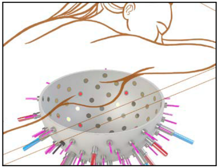
more from more:
multi-spectral, multi-modal, high resolution

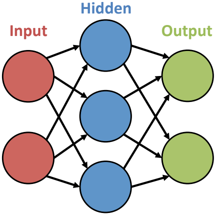
same from less:
low-dose, limited-view, compressed, dynamic

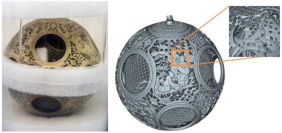
break the routine:
real-time, adaptive, explorative

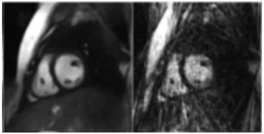
uncertainty quantification & quantitative imaging

machine learning:
embedding, networks for 3D/4D, clinical training data



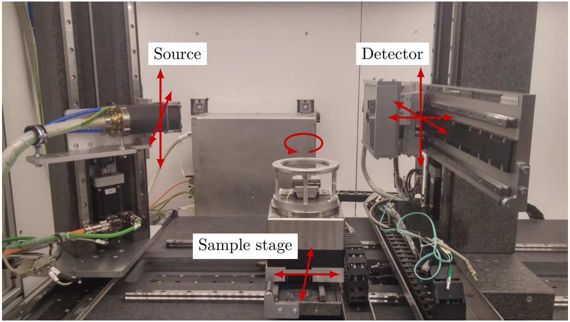






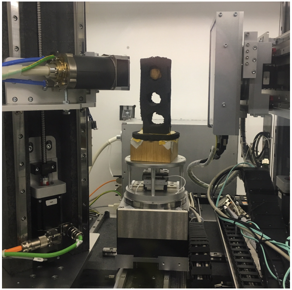
Examples

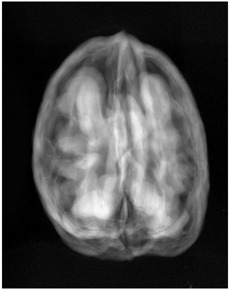
FleX-ray Lab

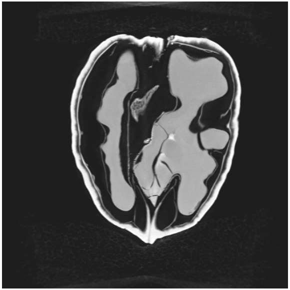


- custom-built, fully-automated, highly flexible
- **Aim: Proof-of-concept** experiments directly accessible to mathematicians and computer scientists.

X-Ray Scan of Static Object





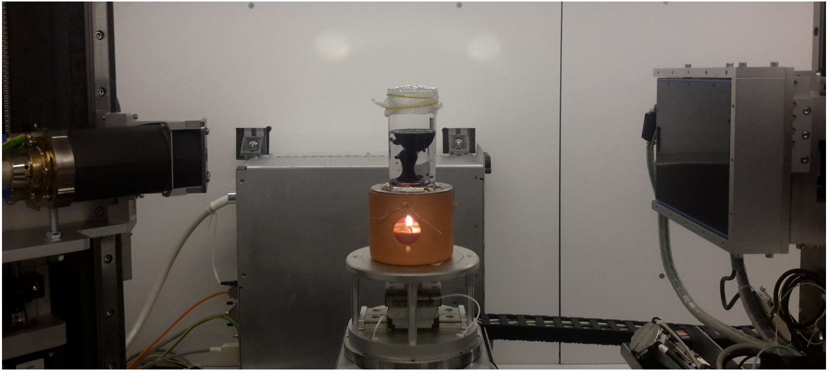


We share

- data sets on zenodo.org, community "CI-CWI"
- open data processing and reconstruction software:
  astra-toolbox.com, github.com/cicwi

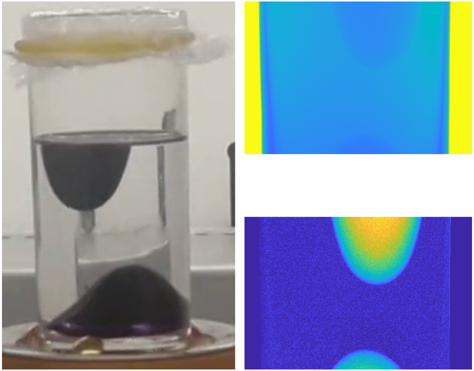
Der Sarkissian, L, van Eijnatten, Colacicco, Coban, Batenburg, 2019.
A Cone-Beam X-Ray CT Data Collection Designed for Machine Learning,
Scientific Data 6, 215 (2019).

X-Ray Scan of Dynamic Object



- canonical example of temperature-driven **two-phase flow instability**
- 120 projections per rotation $\rightarrow$ each projection averaged over 3°
- 40ms exposure per projection $\rightarrow$ 4.8s per rotation

X-Ray Scan of Dynamic Object



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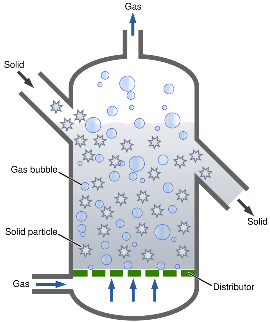
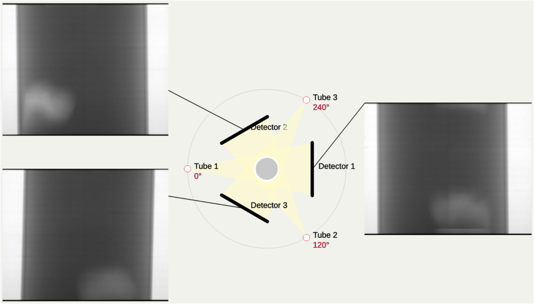
Example: Fluidized Bed Reactors

Collaboration with the Transport Phenomena group at TU Delft.





Overview Dynamic Imaging

Applications

- scientific, industrial and clinical
- vast range of dynamics (rigid motion, elastic deformation, fluid dynamics, crack formation, chemical kinetics, granular flows, ...)

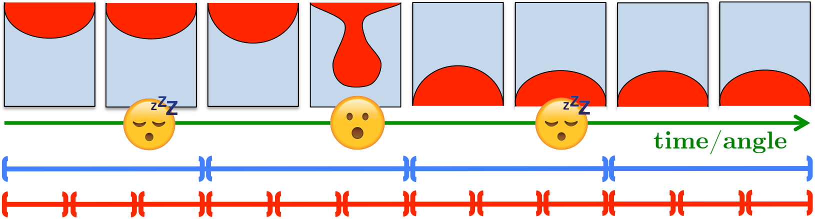
Goals

- motion compensation
- gating
- full dynamic reconstruction (+ simultanous motion estimation?)
- parameter identification in dynamical systems

Challenges

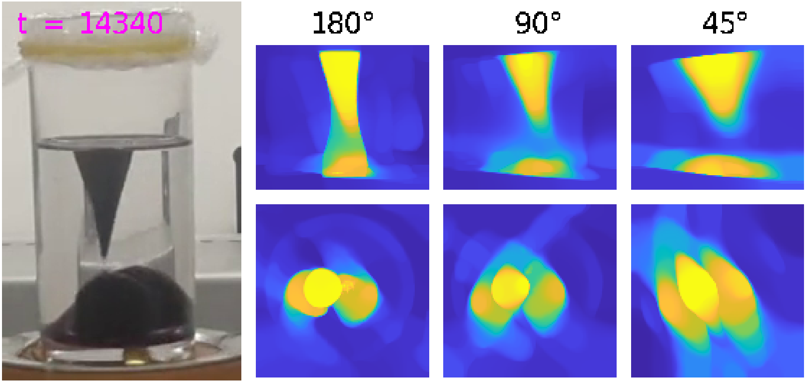
- dynamics too fast for high quality frame-by-frame reconstruction (motion artefacts, noise, low angular res,...)
- mathematical modeling of dynamics
- computational image reconstruction

4D Image Reconstruction Challenges



- binning:
  - large bins $\longrightarrow$ motion artifacts
  - small bins $\longrightarrow$ undersampling /limited view
- 4D is computationally heavier than 3D series
- No "golden bullet": different dynamics, different methods

Lava Lamp: Frame-by-Frame Reconstruction



Joint Image Reconstruction and Motion Estimation

reconstruct image sequence u and motion fields v simultaneously

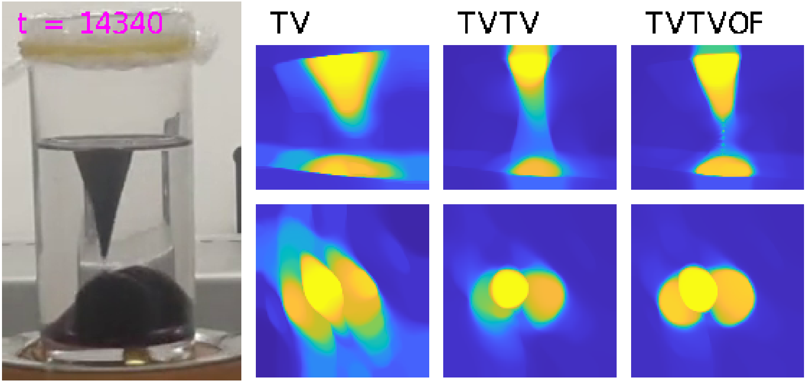
$$\min_{u,v} \sum_t \|A_t\, u_t - f_t\|_2^2 + \mathcal{J}(u_t) + \mathcal{M}(u, v) + \mathcal{H}(v)$$

- data discrepancy
- motion model (PDE)
- spatial assumptions on image
- spatial assumptions on motion

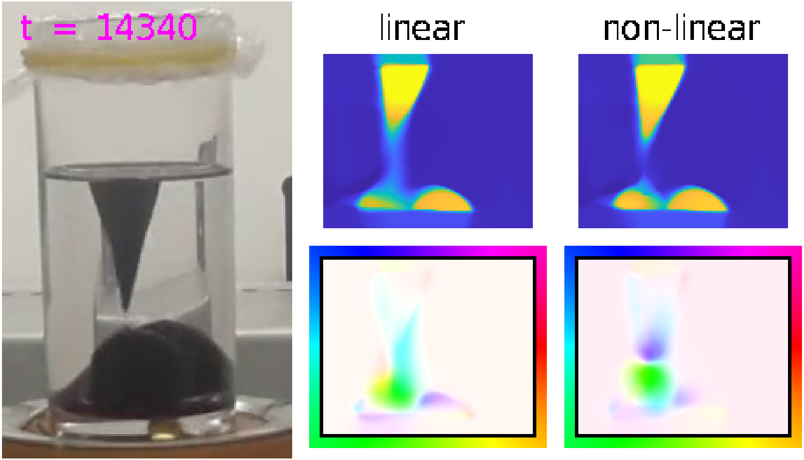
numerical optimization

- alternate between image reconstruction and motion estimation
- image reconstruction **convex but non-smooth**
  primal-dual ("Chambolle-Pock"), augmented Lagrangian ("ADMM")
- motion estimation difficult, **non-convex, non-smooth**
  multi-resolution schemes (pyramids) with linearizations

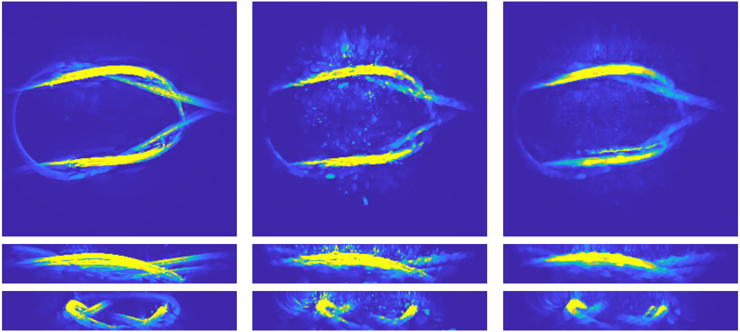
Lava Lamp: Spatio-Temporal Reconstruction



Lava Lamp: Image and Motion Estimation



Dynamic Compressed Sensing for Photoacoustic Tomography



X maxIP

Y maxIP

Z maxIP

full data, TV-fbf | 16x, TV-fbf | 16x, TVTVL2

- **compressed sensing** data acquisition
- evaluation on experimental phantoms and in-vivo recordings

L, Huynh, Betcke, Zhang, Beard, Cox, Arridge, 2018. Enhancing Compressed Sensing 4D Photoacoustic Tomography by Simultaneous Motion Estimation, *SIAM Journal on Imaging Sciences 11:4, 2224-2253.*

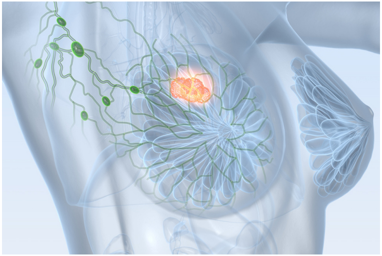
Dynamic Compressed Sensing via Deep Learning

Hauptmann, Arridge, L, Muthurangu, Steeden, 2018. Real-time cardiovascular MR with spatio-temporal artifact suppression using deep learning - proof of concept in congenital heart disease, *Magnetic Resonance in Medicine.*

Motivation: Breast Cancer Imaging

Most common cause of cancer death in women worldwide.

- 25% of all cancer cases in women
- 15% of all cancer deaths in women

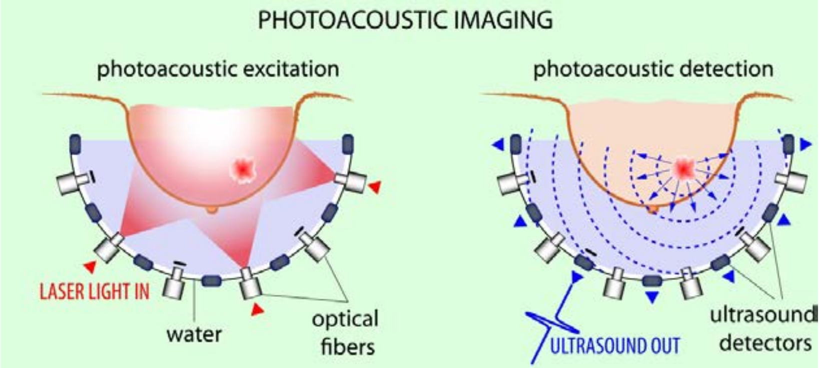


Despite advances in early detection and diagnosis:

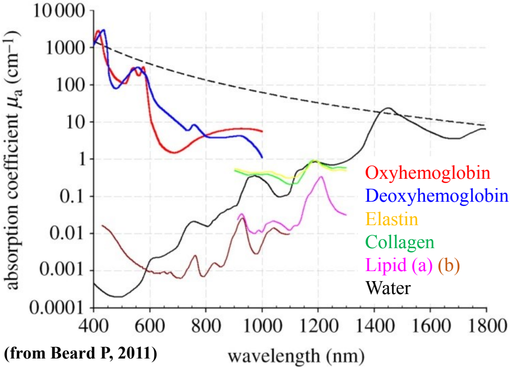
Urgent need for novel imaging techniques providing higher specificity, contrast and image resolution than X-ray mammography at lower costs than MRI.

Quantitative Photoacoustic Breast Imaging

- hybrid imaging: "light in, sound out"
- non-ionizing, near-infrared radiation
- quantitative images of optical properties
- novel diagnostic information



Photoacoustic Imaging: Spectral Properties

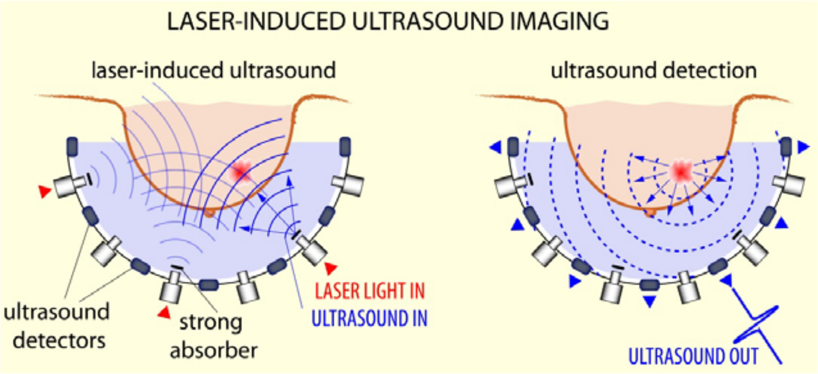


(from Beard P, 2011)

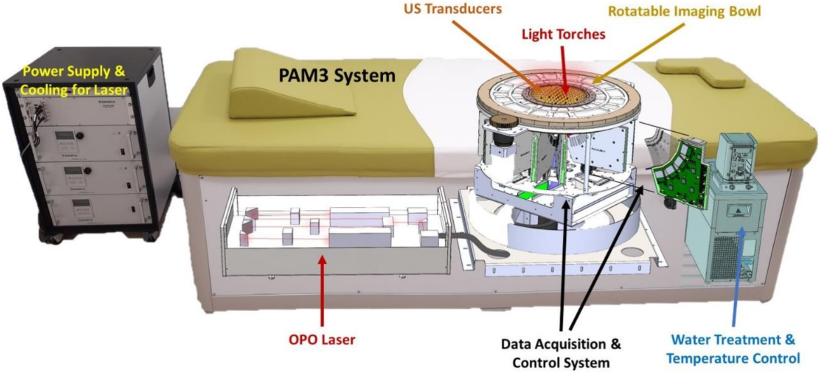
- different wavelengths allow quantitative spectroscopic examinations.
- gap between oxygenated and deoxygenated blood.
- use of contrast agents for molecular imaging.

Quantitative Ultrasonic Breast Imaging

- "sound in, sound out"
- different from conventional US but as safe
- quantitative images of acoustic properties
- novel diagnostic information



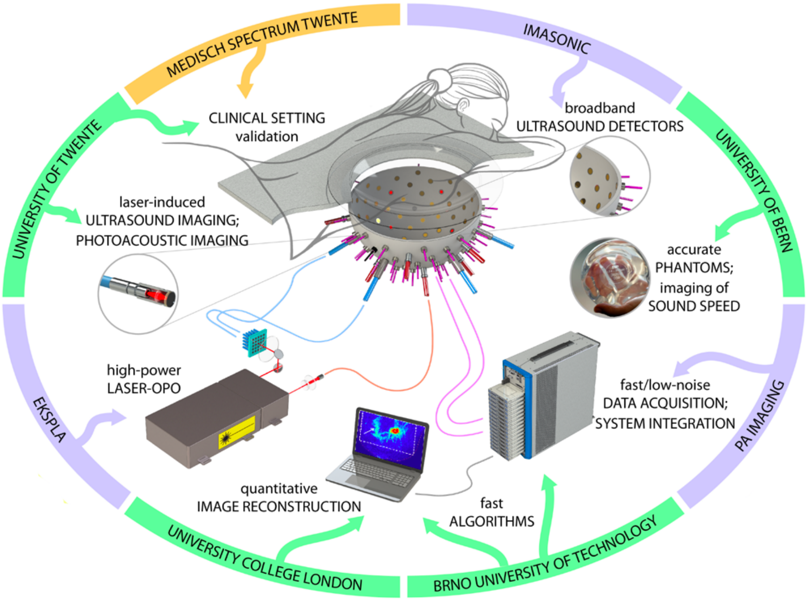
Photoacoustic and Ultrasonic Mammography Scanner



Aim: novel diagnostic information from high resolution maps of optical and acoustic properties

- 512 US transducers on rotatable half-sphere
- 40 optical fibers for photoacoustic excitation

Partners in H2020 Project



Mathematical Modelling (simplified)

Quantitative Photoacoustic Tomography (QPAT)

radiative transfer equation (RTE) + acoustic wave equation

$$(v \cdot \nabla + \mu_a(x) + \mu_s(x))\,\phi(x,v) = q(x,v) + \mu_s(x)\int \Theta(v,v')\phi(x,v')dv',$$

$$p^{PA}(x,t=0) = p_0 := \Gamma(x)\mu_a(x)\int \phi(x,v)dv, \qquad \partial_t p^{PA}(x,t=0) = 0$$

$$(c(x)^{-2}\partial_t^2 - \Delta)p^{PA}(x,t) = 0, \qquad f^{PA} = Mp^{PA}$$

Ultrasound Tomography (UST)

$$(c(x)^{-2}\partial_t^2 - \Delta)p_i^{US}(x,t) = s_i(x,t), \qquad f_i^{US} = M_i p_i^{US}, \qquad i = 1,\dots,n_s$$

Step-by-step inversion

1. $f^{US} \to c$: acoustic parameter identification from boundary data.
2. $f^{PA} \to p_0$: acoustic initial value problem with boundary data.
3. $p_0 \to \mu_a$: optical parameter identification from internal data.

UST Reconstruction Approaches

$$(c(x)^{-2}\partial_t^2 - \Delta)p_i(x,t) = s_i(x,t), \qquad f_i = M_i p_i, \qquad i = 1, \dots, n_{src}$$

Travel time tomography (TTT): geometrical optics approximation.

- ✓ robust & computationally efficient
- **!** valid for high frequencies (attenuation!), low res, lots of data

Reverse time migration (RTM): forward wavefield correlated in time with backward wavefield (adjoint wave equation) via imaging condition.

- ✓ 2 wave simulations, better quality than TTT.
- **!** approximation, needs initial guess, quantitative errors

Full waveform inversion (FWI): fit full model to all data.

- ✓ high res from little data, transducer modelling, constraints
- **!** many wave simulations, complex numerical optimization
- low TRL but already used in 2D systems

time domain vs frequency domain methods

3D Time Domain FWI for Breast USCT

Starting point:

Pérez-Liva, Herraiz, Udías, Miller, Cox, Treeby 2017. Time domain reconstruction of sound speed and attenuation in ultrasound computed tomography using full wave inversion, *JASA*.

Challenges and solutions for 3D:

- ! 2 x n_{src} wave simulations per gradient
- ! computationally & stochastically efficient gradient estimator
- ! memory requirements of gradient computation
- ! slow convergence and local minima
- ! computational resources

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  $\longrightarrow$ **coarse-to-fine multigrid schemes**
- ! computational resources

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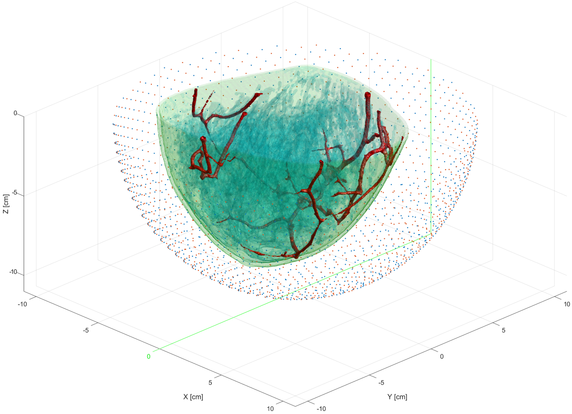
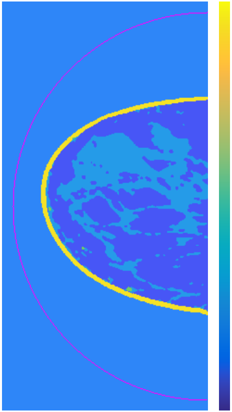
  $\longrightarrow$ **time-reversal based gradient computation**
- ! slow convergence and local minima
  $\longrightarrow$ **coarse-to-fine multigrid schemes**
- ! computational resources
  $\longrightarrow$ **runs on single GPU, can utilize multiple GPUs**

3D FWI: Setup

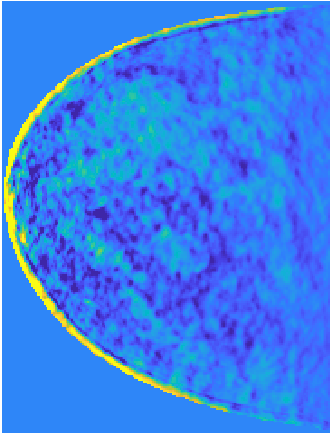




- color range 1435-1665 m/s
- 3D breast phantom at 0.5mm resolution, 1024 sources and receivers
- $442 \times 442 \times 222$ voxel, 3912 time steps

Yang Lou et al. Generation of anatomically realistic numerical phantoms for photoacoustic and ultrasonic breast imaging, *JBO, 2017.*

Starting point in 24h on desktop with single GPU



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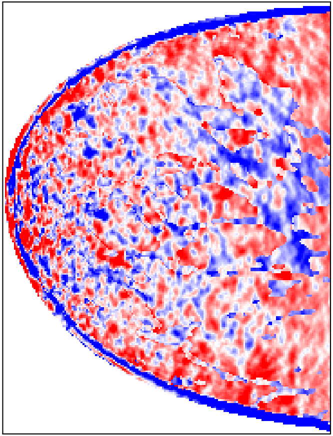
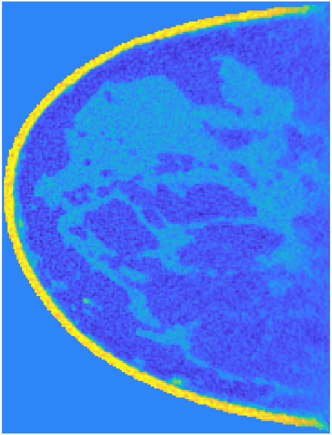


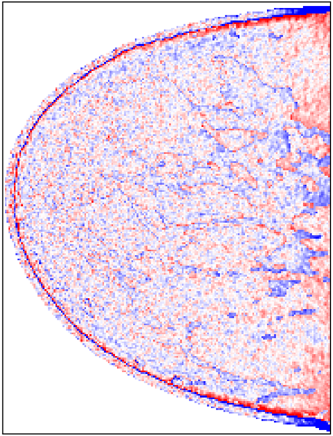
color range -50 to +50 m/s

- single grid
- SGD
- normal single source gradient estimator

3D FWI in 24h on desktop with single GPU



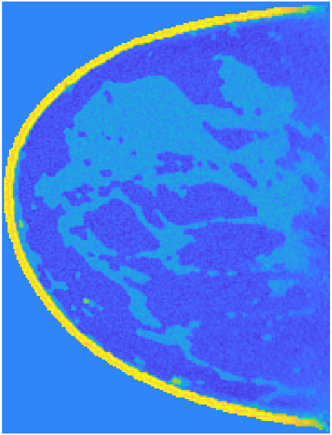
color range 1435 to 1665 m/s



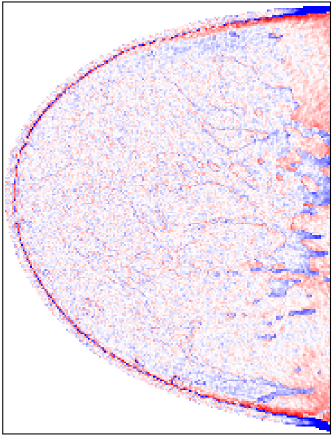
color range -50 to +50 m/s

- multi-grid with 3 level, coarsening factor 2
- SL-BFGS, slowness transform, prog. iter averaging
- time-reversal based source encoding gradient estimator

3D FWI in 24h on cluster with 4 GPU



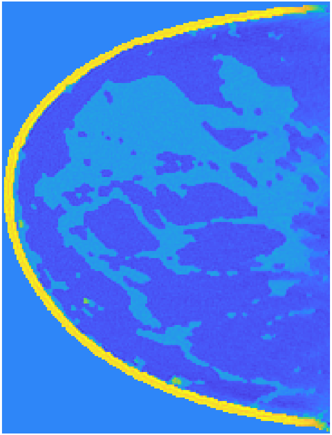
color range 1435 to 1665 m/s



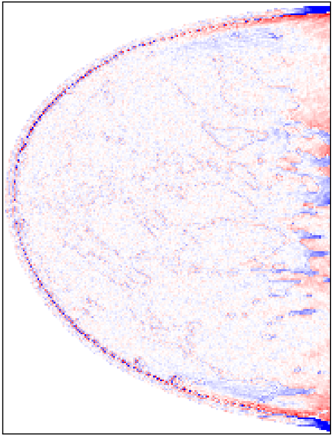
color range -50 to +50 m/s

- multi-grid with 3 level, coarsening factor 2
- SL-BFGS, slowness transform, prog. iter averaging
- time-reversal based source encoding gradient estimator

3D FWI in 24h on cluster with 16 GPU



color range 1435 to 1665 m/s



color range -50 to +50 m/s

- multi-grid with 3 level, coarsening factor 2
- SL-BFGS, slowness transform, prog. iter averaging
- time-reversal based source encoding gradient estimator

Summary

- imaging has broad range of applications
- mathematically: **inverse problem** of reconstructing distributed quantities from indirect observations
- **mathematical modeling**, (solving) **PDEs**, **numerical optimization**
- **challenges:** large-scale, optimization, uncertainty quantification, compressed sensing, dynamic/spectral imaging
- stable solution requires **a-priori information**
- hot topic: **deep learning**

Thank you for your attention!

- **Der Sarkissian, L, van Eijnatten, Colacicco, Coban, Batenburg, 2019.** A Cone-Beam X-Ray CT Data Collection Designed for Machine Learning, *Scientific Data.*
- **Hauptmann, Arridge, L, Muthurangu, Steeden, 2018.** Real-time cardiovascular MR with spatio-temporal artifact suppression using deep learning - proof of concept in congenital heart disease, *Magnetic Resonance in Medicine.*
- **L, Huynh, Betcke, Zhang, Beard, Cox, Arridge, 2018.** Enhancing Compressed Sensing 4D Photoacoustic Tomography by Simultaneous Motion Estimation, *SIAM Journal on Imaging Sciences.*
- **L, Pérez-Liva, Treeby, Cox, 2021.** High Resolution 3D Ultrasonic Breast Imaging by Time-Domain Full Waveform Inversion, *Inverse Problems.*