

Hierarchical Fully-Bayesian Inference for EEG and MEG

"Matti Hämäläinen is visiting us - Workshop"



Felix Lucka

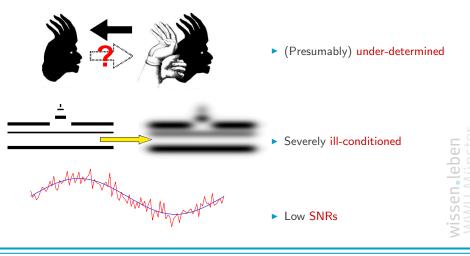
12.10.2012

Background





Reconstruction of brain activity by non-invasive measurement of induced electromagnetic fields?





Summary: The inverse problem is severely ill-posed.

Measurements alone are insufficient and unsuitable to determine solution.

- Incorporation of a-priori information about the solution in an explicit or implicit way:
 - Knowledge about general/specific brain activity?
 - Integration of multimodal information (fMRI, DW-MRI, PET, NIRS)?
 - Mathematical formulation?
 - Computational implementation?
- \implies Variety of inverse methods for EEG/MEG

My focus: Hierarchical Bayesian inference for current density reconstruction (CDR).



Current Density Reconstruction

Lead-field matrix concept:

- L ∈ ℝ^{m×n}; columns represent measurements at *m* sensors caused by the *n* single current dipoles.
- Linear combination of the dipoles is represented by source vector $s \in \mathbb{R}^n$.
- Measurements $b \in \mathbb{R}^m$ caused by *s* can then be calculated via:

$$b = Ls$$



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$$b = Ls$$

Infer s from b? Apparently ill-posed problem:

- $n \gg m$. $\implies b = Ls$ is under-determined.
- ▶ L inherits the bad condition of the continuous problem.
- Noise $\mathcal{E} \sim \mathcal{N}(0, \sigma^2 \mathrm{Id})$ is added to signal.

Common approaches:

- Variational regularization
- (Hierarchical) Bayesian inference
- Spatial scanning methods/beamforming



- 1. Make stochastic model for the relation between parameters, data and noise.
 - $B = Ls + \mathcal{E}$ b is now random valable B
 - Compute probability density of B given s: $p_{like}(b|s)$ (likelihood)



1. Make stochastic model for the relation between parameters, data and noise: $p_{like}(b|s)$.

2. Supplement information given by the data by a-priori information about the parameters of interest. \longrightarrow Bayesian modeling:

- s is considered to be a random variable itself ($s \rightarrow S$).
- Its distribution $p_{prior}(s)$ reflects a-priori assumptions/knowledge.
- ► Task of the prior: Render the estimation problem well-posed.



1. Make stochastic model for the relation between parameters, data and noise: $p_{like}(b|s)$.

2. Supplement information given by the data by a-priori information about the parameters of interest: $p_{prior}(s)$

3. Merge information before the measurement (prior) with the information gained after performing the measurement (likelihood) by Bayes rule:

$$p_{post}(s|b) = rac{p_{like}(b|s)p_{prior}(s)}{p(b)}$$

- ► Conditional distribution of *S* given *B* is called posterior distribution.
- Represents all information on S given the realization of B = b.
- Complete solution to the inverse problem in Bayesian Inference



1. Make stochastic model for the relation between parameters, data and noise: $p_{like}(b|s)$.

2. Supplement information given by the data by a-priori information about the parameters of interest: $p_{prior}(s)$

3. Merge information before the measurement (prior) with the information gained after performing the measurement (likelihood) by Bayes rule: $p_{post}(s|b)$

- 4. Exploit a-posteriori information by infering point estimates:
 - Maximum a-posteriori-estimate (MAP): ŝ_{MAP} := argmax_{s∈ℝn} p_{post}(s|b). Practically: High-dimensional optimization problem.
 - Conditional mean-estimate (CM): ŝ_{CM} := E [s|b] = ∫_{ℝⁿ} s p_{post}(s|b)ds. Practically: High-dimensional integration problem.



Problem: Brain activity is too complex (or our knowledge is too limited) to be captured in a fixed but sufficiently informative prior.

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- Solution: Let the same data determine the prior used for the inference based on this data!



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Sounds like...





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Sounds like...



...but can be formulated into a consistent, statistical reasoning by adding a new dimension of inference: Hyperparameters and hyperpriors.

 \rightarrow Parametric Empirical Bayesian inference

Top-down construction scheme \rightarrow Hierarchical Bayesian modeling (HBM).

Hierarchical Bayesian Modeling (HBM) for CDR: Overview

- Current trend in all areas of Bayesian inference.
- Further development weighted minimum norm schemes.
- Flexible framework for the construction of complex models with different levels for the embedding of different qualitative and quantitative a-priori information: Spatial, temporal, multimodal, functional, anatomical, neuro-physiological...
- Adds an adaptive, data-driven model redcution element into the estimation.
- Embeds several heuristic approaches into sound mathematical framework.
- Comprises many former EEG/MEG methods like MNE, WMNE, LORETA, sLORETA, FOCUSS, MCE,...
- Offers various new ways of inference: Full-MAP, Full-CM, γ-MAP, S-MAP, VB

David Wipf and Srikantan Nagarajan.

A unified Bayesian framework for MEG/EEG source imaging. Neuroimage, 44(3):947-66, February 2009

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Wanted: A prior promoting focal source activity.

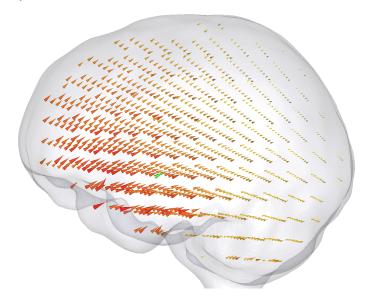
First try:

 Take Gaussian prior with zero mean and covariance Σ_s = γ · Id, γ > 0 (*Minimum norm estimation*).

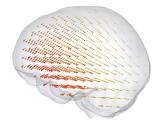
Compute MAP or CM estimate (equal)!

$$\begin{split} \hat{s}_{\text{MAP}} &:= \operatorname*{argmax}_{s \in \mathbb{R}^n} \left\{ \exp\left(-\frac{1}{2\,\sigma^2} \|b - \operatorname{L} s\|_2^2 - \frac{1}{2\gamma} \|s\|_2^2 \right) \right\} \\ &= \operatorname*{argmin}_{s \in \mathbb{R}^n} \left\{ \|b - \operatorname{L} s\|_2^2 + \frac{\sigma^2}{\gamma} \|s\|_2^2 \right\} \end{split}$$

First try: NOT a focal reconstruction.







What went wrong?

- Gaussian variables = characteristic scale given by variance. (not scale invariant)
- All sources have variance $\gamma \Longrightarrow$ Similar amplitudes are likely.
- \implies Focal activity is very unlikely.



- Let sources at single locations i, i = 1, ..., k have different variances γ_i .
- Let the data determine $\gamma_i \implies$ New level of inference!
 - $\gamma = (\gamma_i)_{i=1,...,k}$ are called hyperparameters.
 - Bayesian inference: γ are random variables as well.
 - Their prior distribution $p_{hyper}(\gamma)$ is called hyperprior.



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- Encode focality assumption into hyperprior:
 - Focality: Nearby sources should a-priori not be mutually dependent.
 - Focality: Most sources silent, few with large amplitude;
 - No location preference for activity should be given a priori.



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- Encode focality assumption into hyperprior:
 - γ_i should be stochastically independent.
 - Sparsity inducing hyperprior, e.g., inverse gamma distribution.
 - γ_i should be equally distributed.



In formulas:

$$p_{prior}(s|\gamma) \sim \mathcal{N}(0, \Sigma_{s}(\gamma)), \quad \text{where} \quad \Sigma_{s}(\gamma) = \text{diag}(\gamma_{i} \cdot Id_{3}, i = 1, \dots, k)$$

$$p_{hyper}(\gamma) = \prod_{i=1}^{k} p_{hyper}^{i}(\gamma_{i}) = \prod_{i=1}^{k} p_{hyper}(\gamma_{i}) = \prod_{i=1}^{k} \frac{\beta^{\alpha}}{\Gamma(\alpha)} \gamma_{i}^{-\alpha-1} \exp\left(-\frac{\beta}{\gamma_{i}}\right)$$

 $\alpha > 0$ and $\beta > 0$ determine shape and scale, $\Gamma(x)$ denotes the Gamma function.

Joint prior:
$$p_{pr}(s,\gamma) = p_{prior}(s|\gamma) p_{hyper}(\gamma)$$

Implicit prior: $p_{pr}(s) = \int p_{prior}(s|\gamma) p_{hyper}(\gamma)d\gamma$
 $= \int \mathcal{N}(0, \Sigma_s(\gamma)) p_{hyper}(\gamma)d\gamma \quad \rightsquigarrow \text{``Gaussian scale mixture''}$

(actually a Student's t-distribution with 2(lpha+1) degrees of freedom)

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Posterior, general:

 $p_{\textit{post}}(s, \gamma | b) \propto p_{\textit{like}}(b | s) p_{\textit{prior}}(s | \gamma) p_{\textit{hyper}}(\gamma)$ Comparison: $p_{\textit{post}}(s | b) \propto p_{\textit{like}}(b | s) p_{\textit{prior}}(s)$

Posterior, concrete:

$$p_{post}(s, \gamma | b) \propto \exp\left(-\frac{1}{2\sigma^2} \|b - Ls\|_2^2 - \sum_{i=1}^k \left(\frac{\frac{1}{2} \|s_{i*}\|^2 + \beta}{\gamma_i} + \left(\alpha + \frac{5}{2}\right) \ln \gamma_i\right)\right)$$

Analytical advantages...

- Energy is quadratic with respect to s
- Factorizes over γ_i 's.

and disadvantages...

• Energy is non-convex w.r.t. (s, γ) (posterior is multimodal)

Excursus: Full-, Semi-, and Approximate Inversion

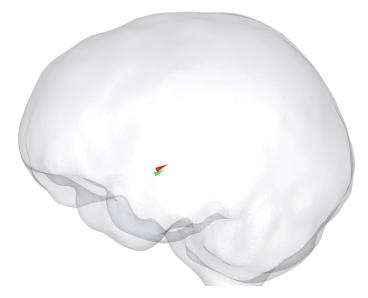
Two types of parameters \longrightarrow more possible ways of inference.

- Full-MAP: Maximize $p_{post}(s, \gamma | b)$ w.r.t. s and γ .
 - Full-CM: Integrate $p_{post}(s, \gamma | b)$ w.r.t. s and γ .
 - γ -MAP: Integrate $p_{post}(s, \gamma|b)$ w.r.t. s, and maximize over γ , first. Then use $p_{post}(s, \hat{\gamma}(b)|b)$ to infer s. (Hyperparameter MAP/Empirical Bayes)
 - S-MAP: Integrate $p_{post}(s, \gamma | b)$ w.r.t. γ , and maximize over s.
 - VB: Assume approximative factorization $p_{post}(s, \gamma|b) \approx \hat{p}_{post}(s|b) \hat{p}_{post}(\gamma|b)$; Approximate both with distributions that are analytically tractable.

Focus of our work: Fully Bayesian inference.

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Full-MAP estimate





Starting Point for our Studies

- A specific HBM aims to recover source configurations consisting of few, focal sources (introduced in Sato et al., 2004; further examined in Nummenmaa et al., 2007; Wipf and Nagarajan, 2009; Calvetti et al., 2009)
- Calvetti et al., 2009 found promising first results for certain inference strategies for deep-lying sources and the separation of multiple (focal) sources.

Limitations of Calvetti et al., 2009 :

- Specific results were not convincing; reason unclear.
- No systematic examination; only two source scenarios.
- Head models insufficient.



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Why are we interested in that?

(rhetorical question to switch to another talk in an elegant way...)

Tasks and Problems for EEG/MEG in Presurgical Epilepsy Diagnosis

EEG/MEG in epileptic focus localization:

- Focal epilepsy is believed to originate from networks of focal sources.
- Active in inter-ictal spikes.
- Task 1: Determine number of focal sources (multi focal epilepsy?).
- Task 2: Determine location and extend of sources.

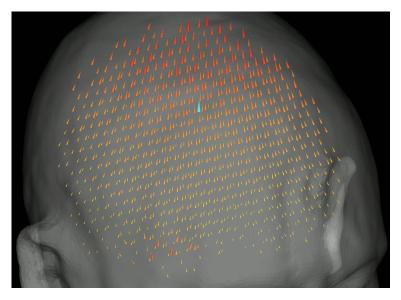
Unknown number and spatial extend of sources? \rightarrow Current density reconstruction (CDR).

Problems of established CDR methods:

- **Depth-Bias**: Reconstruction of deeper sources too close to the surface.
- Masking: Near-surface sources "mask" deep-lying ones.

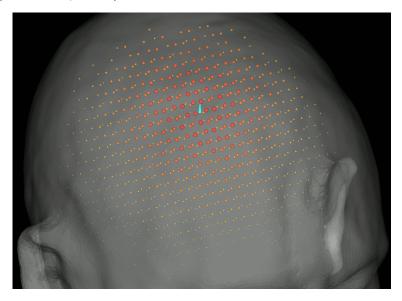
Depth Bias: Illustration

One deep-lying reference source (blue cone) and minimum norm estimate (MNE, Hämäläinen and Ilmoniemi, 1994).



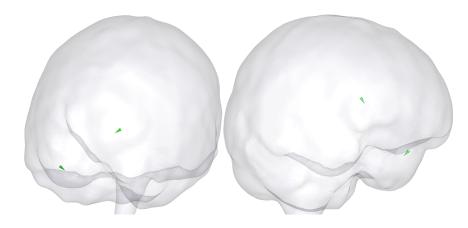
Depth Bias: Illustration

One deep-lying reference source (blue cone) and sLORETA result (Pascual-Marqui, 2002).



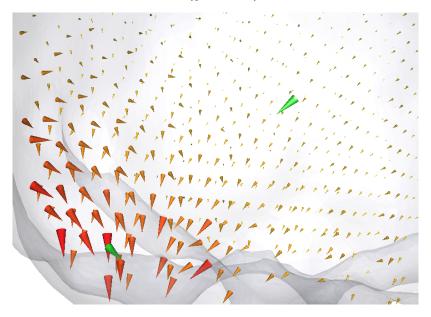
Masking: Illustration

Reference sources.



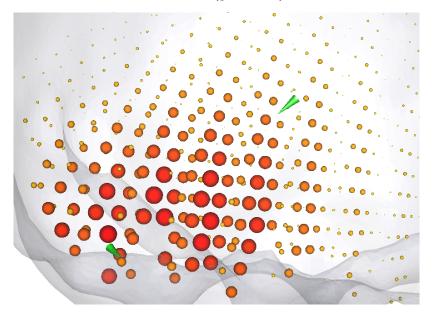
Masking: Illustration

MNE result and reference sources (green cones).



Masking: Illustration

sLORETA result and reference sources (green cones).





Diploma Thesis and Neuroimage Paper (EEG only!)

Key question

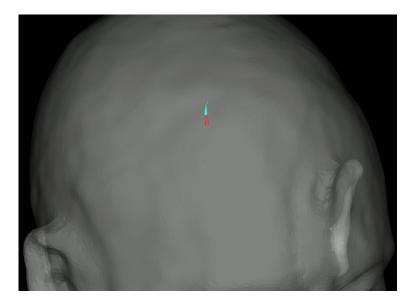
Can Full-MAP and Full-CM for HBM overcome the limitations (depth-bias, masking) of established CDR methods?

Work program:

- Implementation of Full-MAP and Full-CM inference for HBM with realistic, high resolution Finite Element (FE) head models.
- Propose own algorithms for Full-MAP estimation.
- Introduction of suitable performance measures for validation of simulation studies.
- Systematic examination of performance concerning depth-bias and masking in simulation studies.

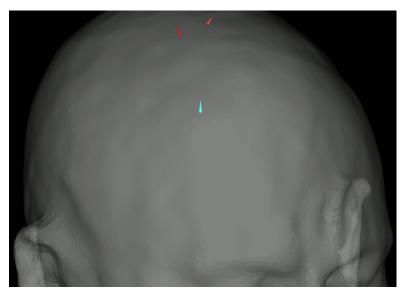
Results Depth Bias: Illustration

One deep-lying reference source (blue cone) and Full-CM result.



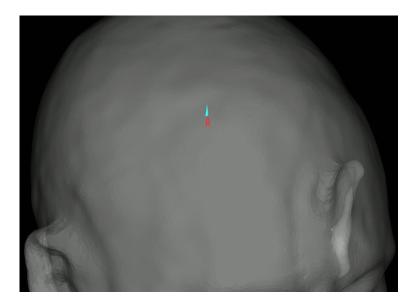
Results Depth Bias: Illustration

One deep-lying reference source (blue cone) and Full-MAP result proposed by Calvetti et al., 2009.



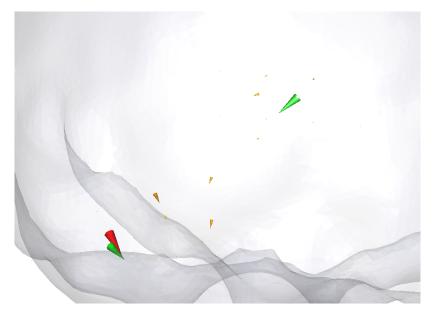
Results Depth Bias: Illustration

One deep-lying reference source (blue cone) and Full-MAP result proposed by us.



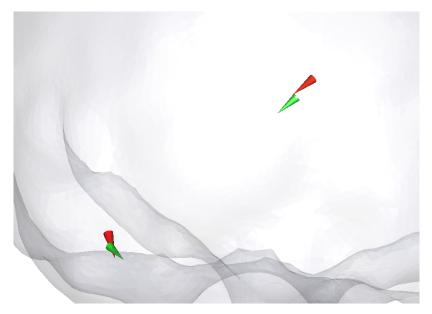
Results Masking: Illustration

Full-CM result and reference sources (green cones).



Results Masking: Illustration

Full-MAP result (by our algorithm) and reference sources (green cones).





Systematic Studies: Summary

Study 1 (depth-bias):

- ▶ Reconstruction of single 1000 dipoles; random location and orientation.
- Reconstructions were compared using different performance measures.
- Specific examination of depth bias.

Study 2 (masking):

- Reconstruction of 1000 source configurations consisting of one near-surface and one deep-lying dipole.
- Reconstructions were compared using a performance measure based on optimal transport (called earth mover's distance, a Wasserstein metric).



Systematic Studies: Summary

Results for Full-MAP and Full-CM estimation:

- Good performance in all validation measures.
- No depth bias.
- Good results w.r.t. orientation, amplitude and spatial extend.
- Full-MAP estimate (by our algorithm): Best results in every aspect examined.

Full results:

Felix Lucka., Sampsa Pursiainen, Martin Burger, Carsten H. Wolters. 2012. Hierarchical Bayesian Inference for the EEG Inverse Problem using Realistic FE Head Models: Depth Localization and Source Separation for Focal Primary Currents. Neuroimage 61(4), 1364-1382.



Conclusions, Diploma Thesis and Neuroimage paper

Key question

Can Full-MAP and Full-CM for HBM overcome the limitations (depth-bias, masking) of established CDR methods?

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Conclusions, Diploma Thesis and Neuroimage paper

Key question

Can Full-MAP and Full-CM for HBM overcome the limitations (depth-bias, masking) of established CDR methods?

Results

- Hierarchical Bayesian modeling used with realistic head modeling is a promising framework for EEG CDR.
- Promising results for deep sources (no depth bias).
- Promising results for challenging multiple source scenarios (no masking).

 \star A promising tool for the analysis of neurophysiological data. \star

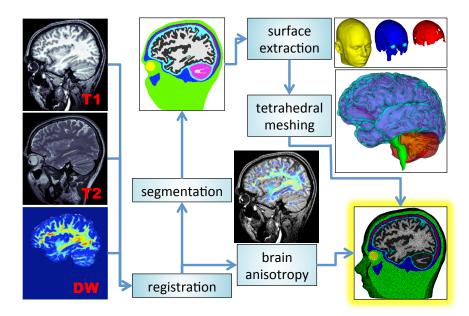




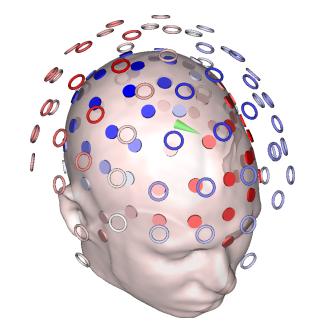
We addressed two questions that were posed in the outlook of the paper:

- ► EEG vs. MEG and EEG/MEG combination (EMEG):
 - Do our findings also apply for MEG?
 - Previously (e.g., Molins et al., 2008), the differences between EEG and MEG have mainly been examined by established inverse methods. How are things for fully-Bayesian inference for HBM?
- Realistic Head Model: Formerly, we used a simplified head model with a homogenous inner brain (to facilitate the interpretation of the results).
 Especially for EEG/MEG combination, the use of a realistic, individual and anisotropic head model is mandatory.

Model Setup: Modeling Pipeline



Model Setup: Sensor Configuration





Simulation Studies

Inverse Methods:

- Three fully-Bayesian HBM methods:
 - Full-MAP estimates
 - Full-CM estimates
 - Full-NM (Near-Mean) estimates
- Minimum norm estimates (MNE) with different weightings (WMNE).
- sLORETA

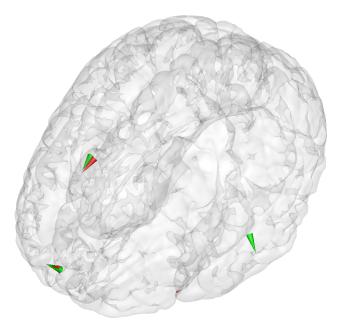
Simulation studies (similar to Neuroimage paper):

- 1. Single dipole recovery \longrightarrow localization, focality, depth-bias.
- 2. Two dipole recovery \longrightarrow source separation.

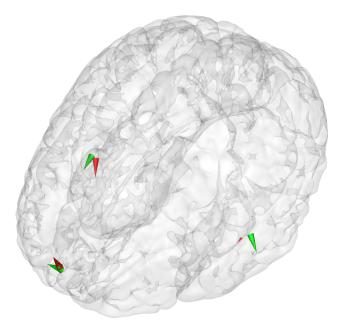
Source configurations were reconstructed using

(a) EEG alone (b) MEG alone (c) EMEG data

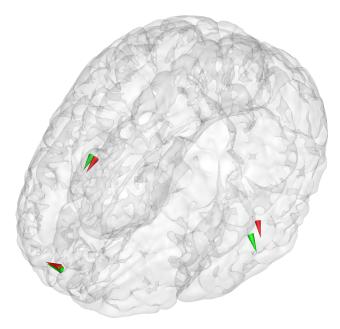
Exemplary Three Dipole Scenarios 1: HBM-NM Estimate, EEG alone



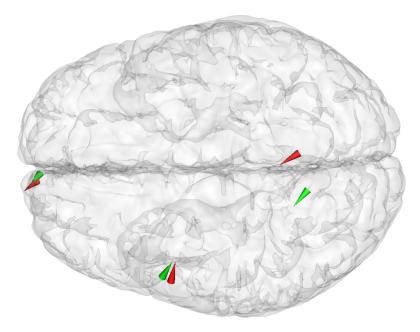
Exemplary Three Dipole Scenarios 1: HBM-NM Estimate, MEG alone



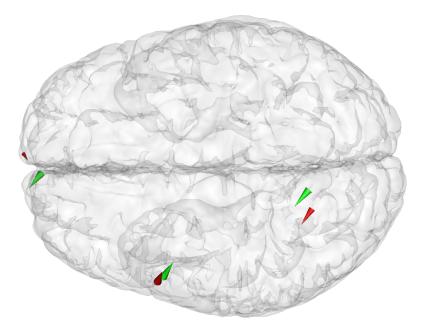
Exemplary Three Dipole Scenarios 1: HBM-NM Estimate, EMEG



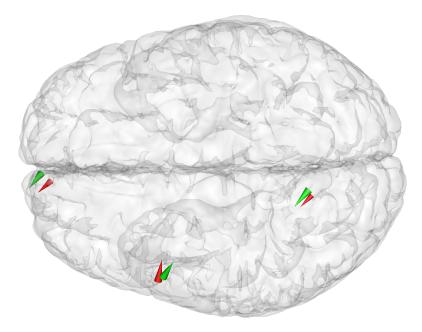
Exemplary Three Dipole Scenarios 2: HBM-NM Estimate, EEG alone



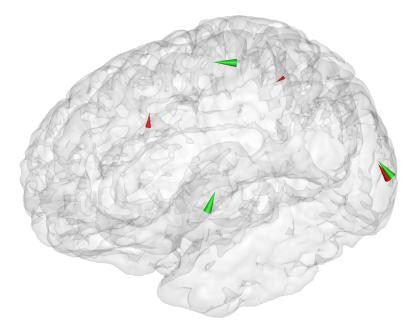
Exemplary Three Dipole Scenarios 2: HBM-NM Estimate, MEG alone



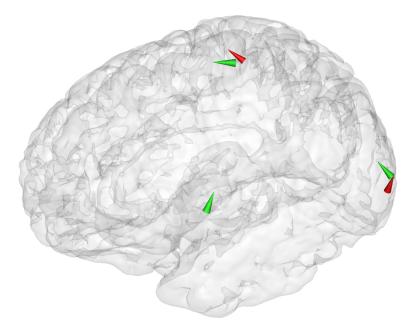
Exemplary Three Dipole Scenarios 2: HBM-NM Estimate, EMEG



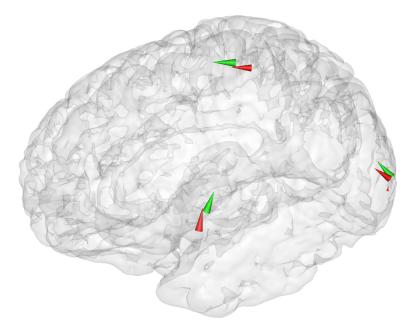
Exemplary Three Dipole Scenarios 3: HBM-NM Estimate, EEG alone



Exemplary Three Dipole Scenarios 3: HBM-NM Estimate, MEG alone



Exemplary Three Dipole Scenarios 3: HBM-NM Estimate, EMEG





Biomag Results and Conclusions, EEG vs. MEG

Results:

- ▶ HBM methods and sLORETA do not show a depth bias in any modality.
- Weighting of MNE to avoid depth bias in all modalities is difficult and comes at the cost of other draw-backs.
- The average localization performance (mean DLE) of HBM methods is equal for EEG and MEG. For WMNE variants and sLORETA, it is better for MEG.
- The mean EMD (localization + spatial extend) is better for EEG than MEG for all methods, although the differences are differently pronounced.



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Conclusions:

- Statements about localization properties of single modalities cannot be made without a reference to the inverse method used. This is a feature of the ill-posed nature of the EEG/MEG inverse problem.
- ► For all MNE variants and sLORETA, MEG offers a better localization (DLE) of single dipoles while having a higher EMD ⇒ better localization comes at the costs of a larger blurring.



Biomag Results and Conclusions, EEG/MEG Combination

Results:

- The combination improves the average performance of all methods (measured in EMD and DLE).
- The improvement of the EMD of HBM methods for multiple source scenarios is larger than for established methods.
- > The combination reduces variance and outliers in the error statistics.
- The depth localization does not always profit from combination, especially if a single modality is very weak in that aspect.



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Conclusions:

- EEG/MEG combination stabilizes and improves source reconstruction to a considerable amount.
- Fully-Bayesian HBM methods profit from EEG/MEG combination especially for source separation in multiple source scenarios. This further underlines the potential of these methods for complex sources scenarios in real applications.

Outlook & Future Work

- Current focus: Processing of real data:
 - combined AEP/AEF and SEF/SEP data;
 - interictal epileptic activity;
- Practical aspects of EEG/MEG combination: Noise rescaling, volume conductor calibration and sensor weighting.
- Temporal extension of our HBM methods.
- Generalization of the specific HBM
 - to also recover more extented source scenarios;
 - to model inhibition, excitation and syncrony between brain areas;
 - to incorporate mulitmodal information from, e.g., DW-MRI, PET, SPECT, fMRI, NIRS;
- Comparison of HBM methods and EEG and MEG for extended source configurations.
- Comparison to other HBM-based methods like variational Bayesian approaches.



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Software used by our group:

- Registration: FSL,FAIR;
- Segmentation: FSL, CURRY;
- FEM Meshing: Tetgen, vgrid, iso2mesh:
- FEM Computation: SimBio;

- Data Preprocessing: CURRY, BESA;
- Inverse computation: Matlab;
- Volume Visualization: SCIRun:
- Everything else & software integration: Matlab;