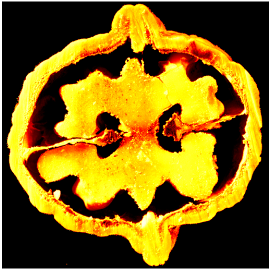
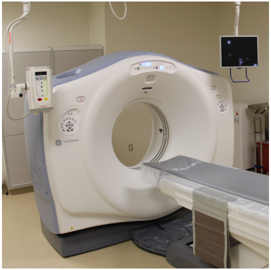
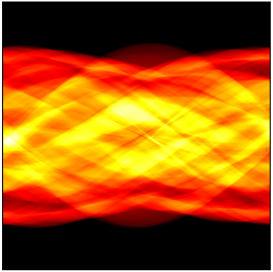
X-Ray Computed Tomography







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Felix.Lucka@cwi.nl

Mastermath Course
"Inverse Problems and Imaging"

March 21, 2022

Centrum Wiskunde & Informatica (CWI)

- National research institute for mathematics and computer science, founded 1946.
- Focus: Fundamental research problems derived from societal needs.
- ∼200 people working in 15 research groups on 4 research themes: Algorithms, Data & Intelligent Systems, Security & Cryptography, Quantum Computing
- National and international industry and academic collaborations.
- 27 spin-off companies
- opportunities for MSc and PhD students

Computational Imaging @ CWI

- headed by Tristan van Leeuwen (also Utrecht Uni), ∼20 members
- mathematics, computer science & (medical) physics
- advanced computational techniques for 3D imaging
- (inter-)national collaborations from science, industry & medicine
- one of the two main developers of the ASTRA Toolbox
- FleX-ray Lab: custom-made, fully-automated X-ray CT scanner linked to large-scale computing hardware

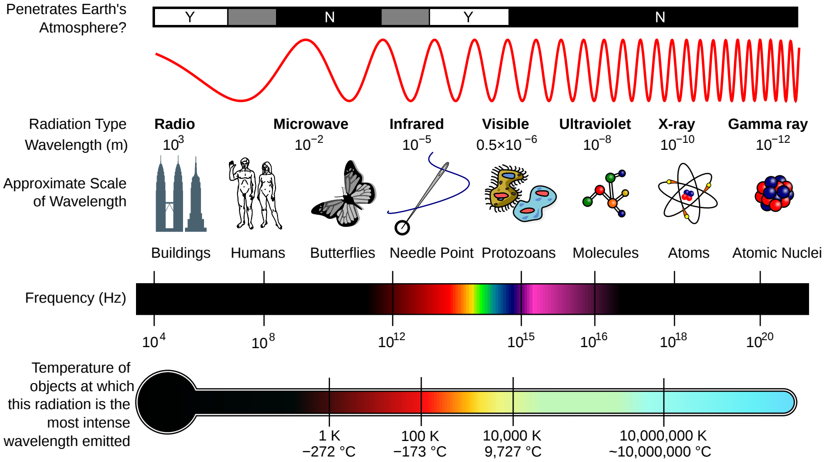
History of X-rays

(a) Wilhelm Röntgen (1845-1923)

(b) First X-ray image (1895)

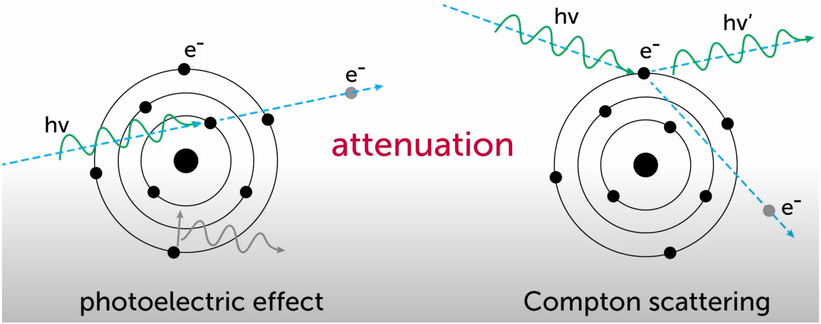
source: Wikimedia Commons

What are X-rays?



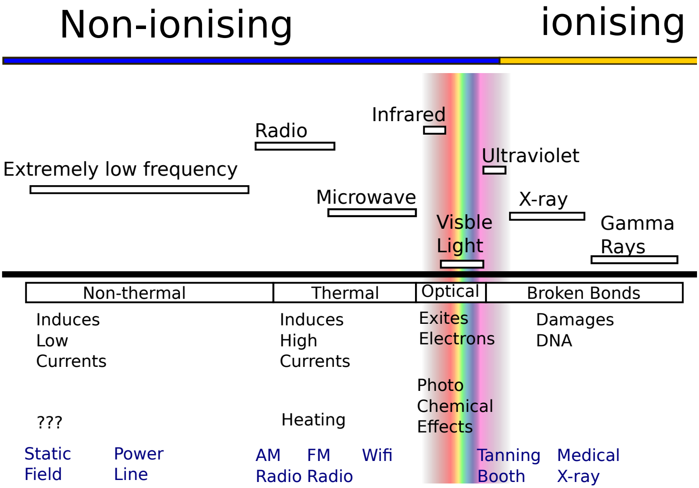
source: Wikimedia Commons

X-ray-matter interaction



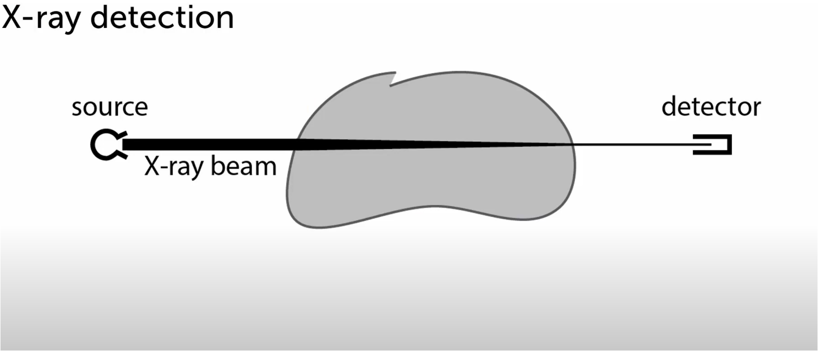
Taken from corresponding video by the ASTRA toolbox team YouTube

How do X-rays interact with materials?



source: Wikimedia Commons

Mathematics of CT 1: Beer's Law



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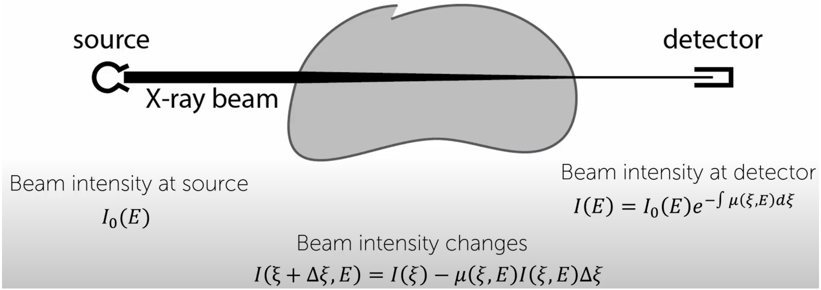
Mathematics of CT 1: Beer's Law

X-ray detection



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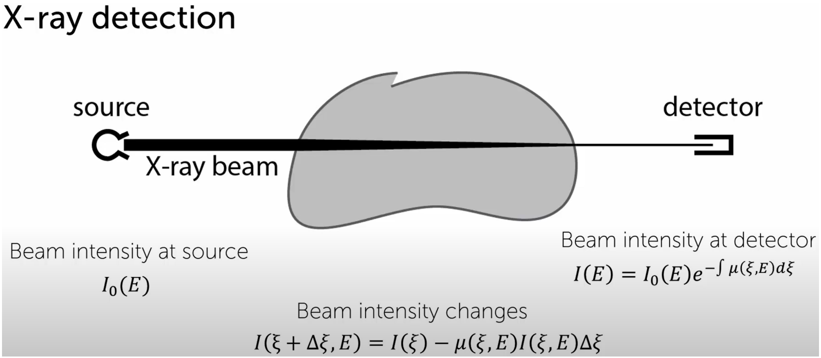
Mathematics of CT 1: Beer's Law



$$\Rightarrow P := -\log\left(\frac{I(E)}{I_0(E)}\right) = \int_{beam} \mu(\xi, E)d\xi$$

Taken from corresponding video by the ASTRA toolbox team YouTube

Radiography

An excellent video by Samuli Siltanen: YouTube

source: Wikimedia Commons

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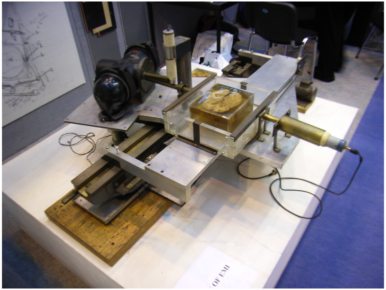
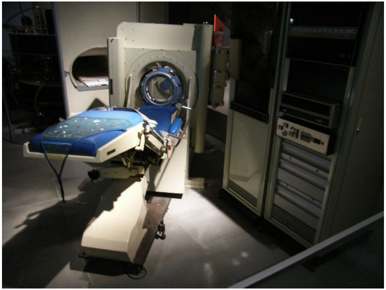
History of Computed Tomography (CT)

Godfrey Hounsfield and Allan McLeod Cormack developed X-ray tomography (Nobelprize 1979)



(c) CT prototype



(d) first comercial CT head scanner

source: Wikimedia Commons

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Modern CT Scanner

a video of a scanner during rotation YouTube

source: Wikimedia Commons

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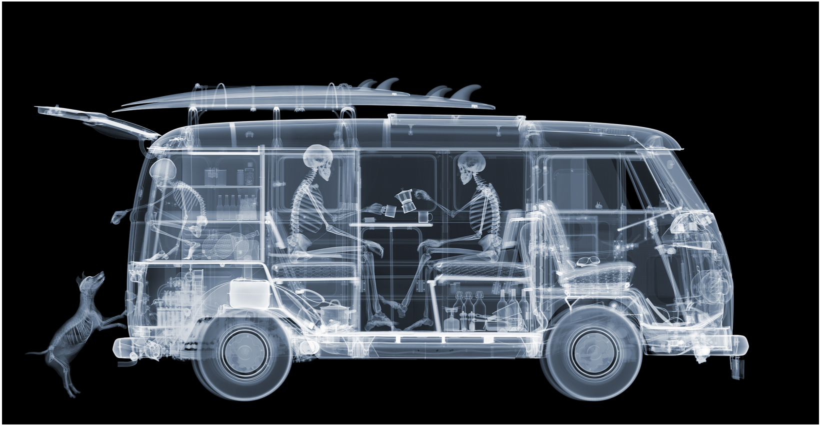
Break & questions time



Nick Veasey, VW Camper Van , 2019

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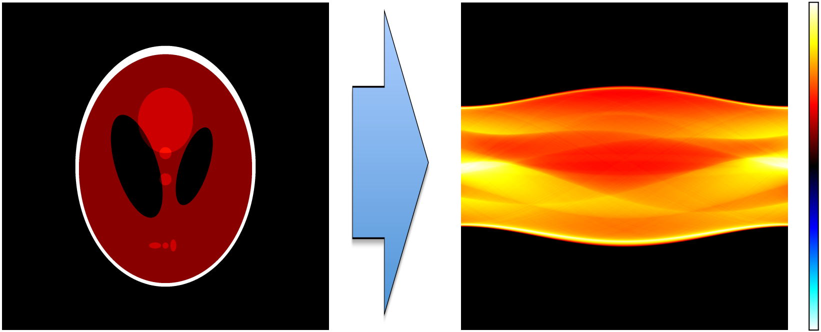
Disclaimer

!!! Notation on these slides varies from the script !!!

From projections to sinograms

Another excellent video by Samuli Siltanen: YouTube



(a) image

(b) sinogram

From projections to sinograms

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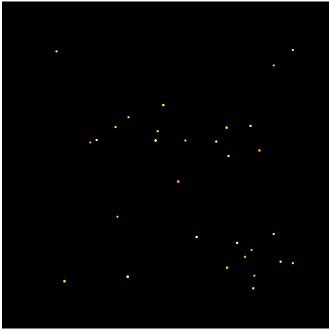
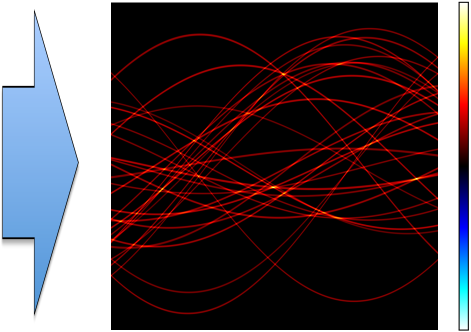


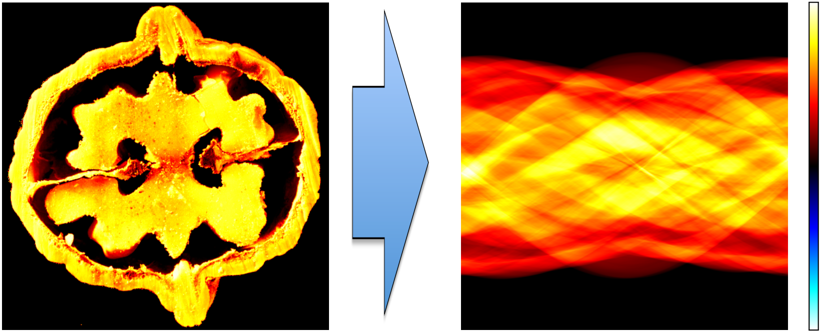
(a) image



(b) sinogram

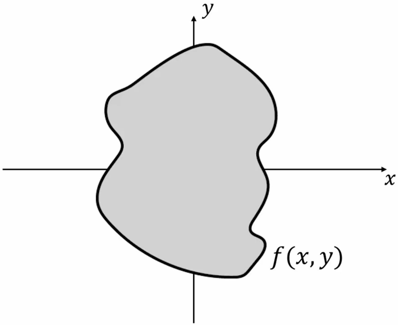
From projections to sinograms

Another excellent video by Samuli Siltanen:



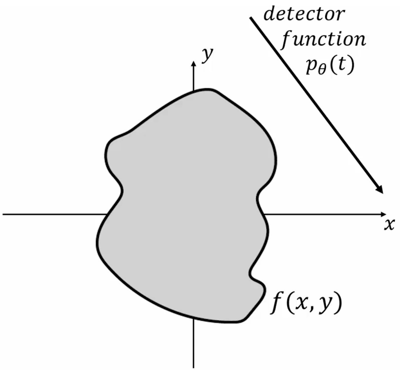
(a) image (b) sinogram

Radon transform



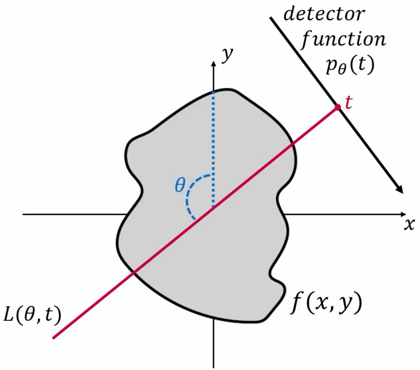
Taken from corresponding video by the ASTRA toolbox team YouTube

Radon transform



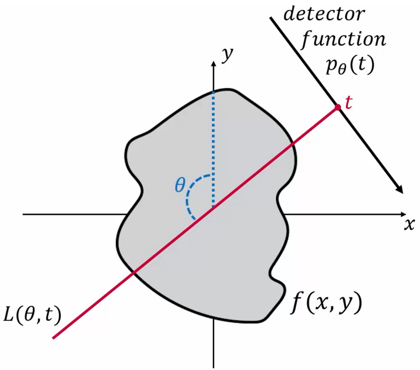
Taken from corresponding video by the ASTRA toolbox team YouTube

Radon transform



Taken from corresponding video by the ASTRA toolbox team YouTube

Radon transform

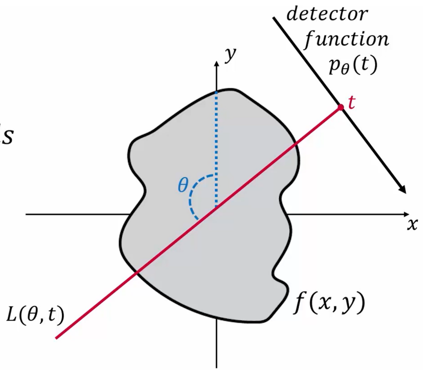


$$L(\theta, t) = \{(x, y) \in \mathbb{R} \times \mathbb{R}: x\cos\theta + y\sin\theta = t\}$$

Taken from corresponding video by the ASTRA toolbox team YouTube

Radon transform

$$p_\theta(t) = \int_{L(\theta,t)} f(x,y)\,ds$$

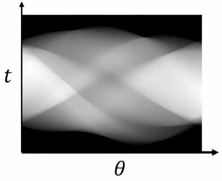


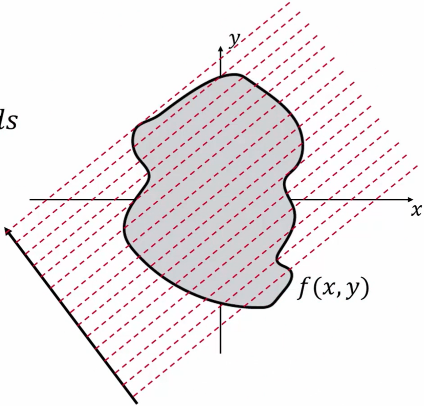
$$L(\theta,t) = \{(x,y) \in \mathbb{R}\times\mathbb{R}: x\cos\theta + y\sin\theta = t\}$$

Taken from corresponding video by the ASTRA toolbox team YouTube

Radon transform

$$\mathcal{R}f(\theta, t) = \int_{L(\theta,t)} f(x,y)\,ds$$



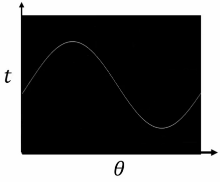


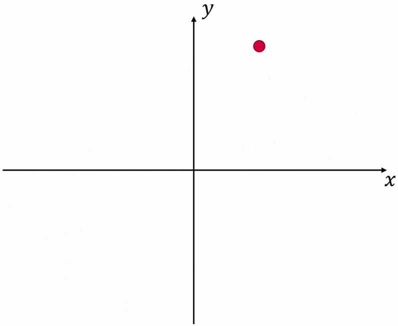
Taken from corresponding video by the ASTRA toolbox team YouTube

Radon transform

$$\boxed{\mathcal{R}f(\theta,t)} = \int_{L(\theta,t)} f(x,y)\,ds$$

Sinogram





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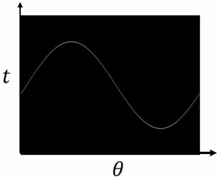
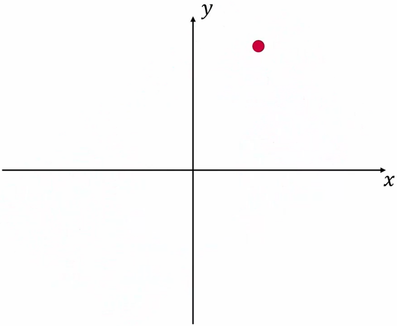
Radon transform

$$\boxed{\mathcal{R}f(\theta,t)} = \int_{L(\theta,t)} f(x,y)\,ds$$

Sinogram





R is a linear operator, but is it invertible?

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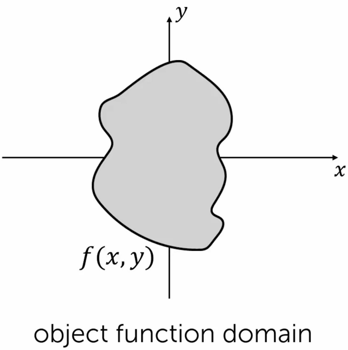
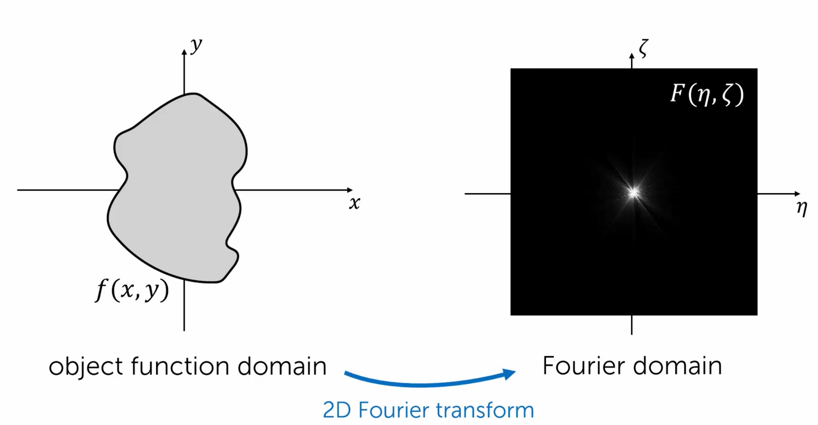
Fourier Slice Theorem



Taken from corresponding video by the ASTRA toolbox team YouTube

Fourier Slice Theorem



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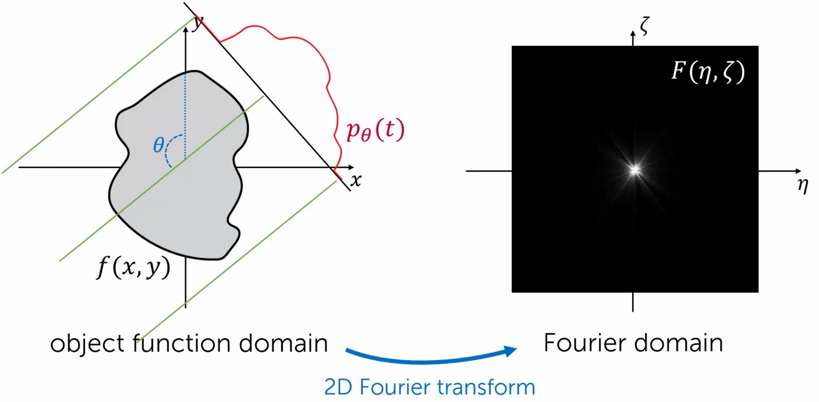
CWI

Fourier Slice Theorem



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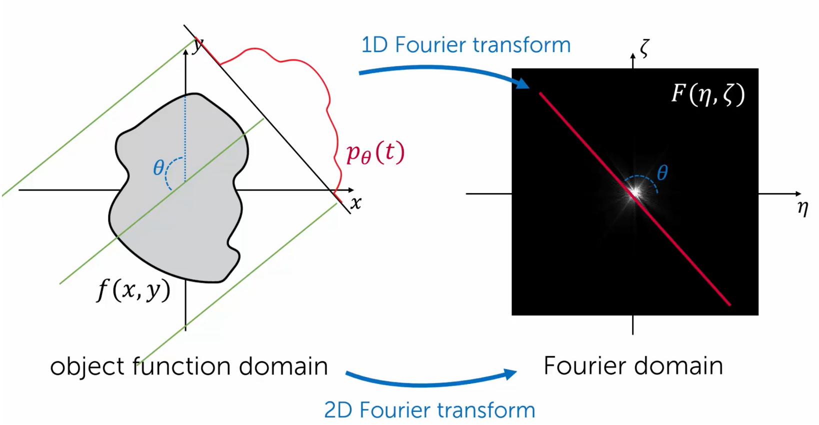
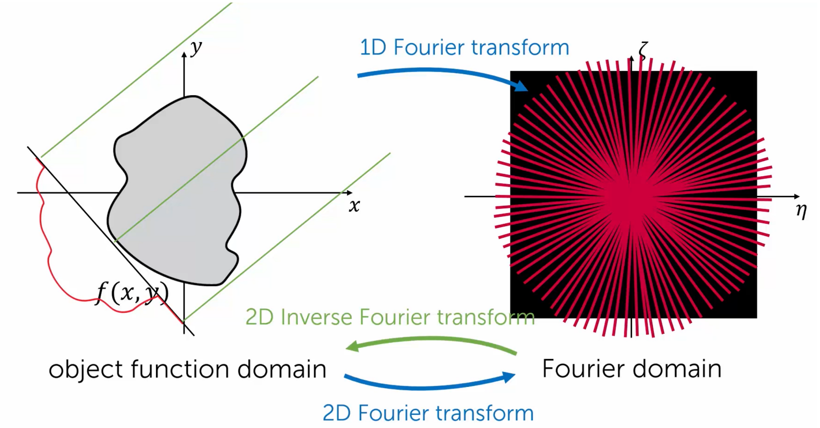
Fourier Slice Theorem



$$\mathcal{F}_1\left[\mathcal{R}_\theta f\right](\omega) = \mathcal{F}_2\left[f\right](\omega\nu), \qquad \nu = (\cos\theta, \sin\theta)$$

Taken from corresponding video by the ASTRA toolbox team YouTube

Fourier Slice Theorem



$$\mathcal{F}_1\left[\mathcal{R}_\theta f\right](\omega) = \mathcal{F}_2\left[f\right](\omega\nu), \qquad \nu = (\cos\theta, \sin\theta)$$

Taken from corresponding video by the ASTRA toolbox team YouTube

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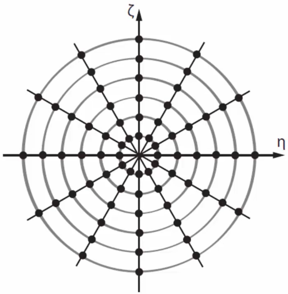
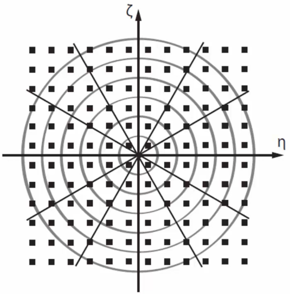
Non-uniform Fourier sampling



Fourier sampling with Fourier Slice Theorem

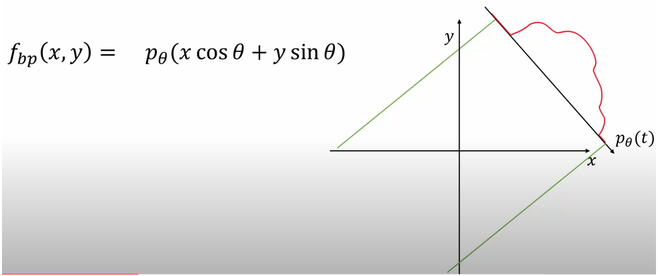


Sampling required by Fast Fourier Transform (FFT)

- ! high frequencies (= high resolution details) undersampled
- ! sampling non-uniform

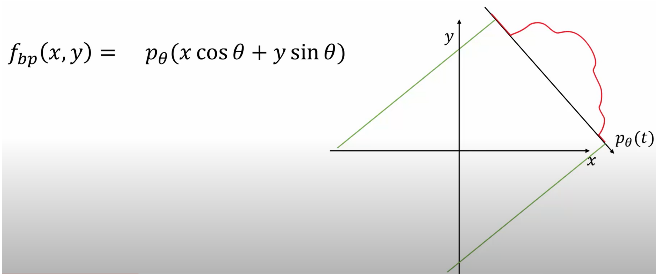
Taken from corresponding video by the ASTRA toolbox team YouTube

Backprojection



$$BP\left[p(\theta, t)\right](x, y) := \int p_\theta\left(x\cos\theta + y\sin\theta\right) d\theta$$

Backprojection



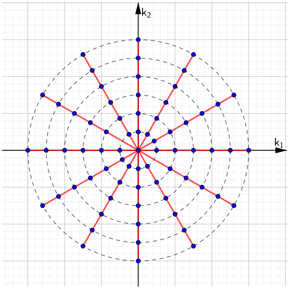
$$BP\left[p(\theta, t)\right](x, y) := \int p_\theta\left(x\cos\theta + y\sin\theta\right) d\theta$$

✓ $BP = \mathcal{R}^*$ and computationally efficient

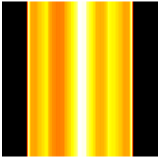
! $\mathcal{R}^*\mathcal{R}\left[f\right] \propto \frac{1}{\|x\|} * f$

Backprojection in action

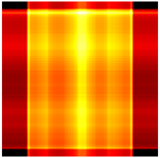
(a) true image



(b) Fourier sampling



(c) 1 angle



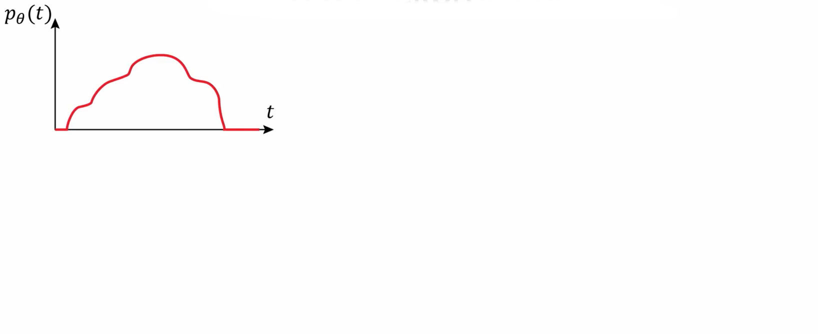
(d) 2 angles

(e) 8 angles

(f) 64 angles

(g) 256 angles

Filtered backprojection



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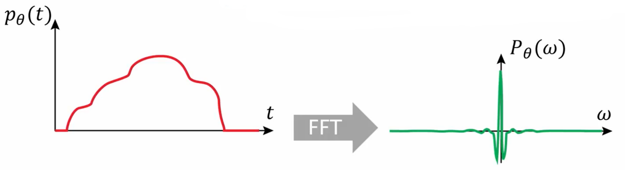
Filtered backprojection



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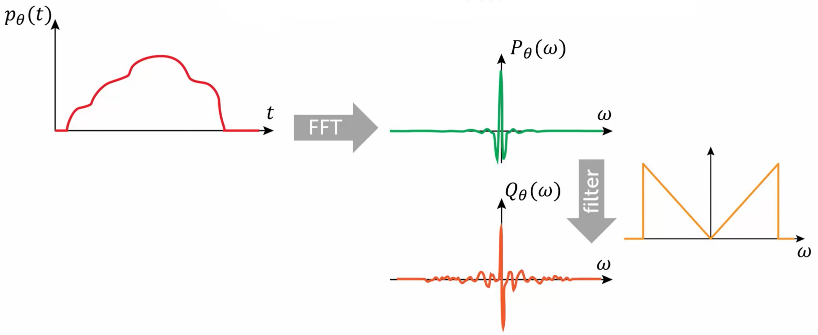
Filtered backprojection



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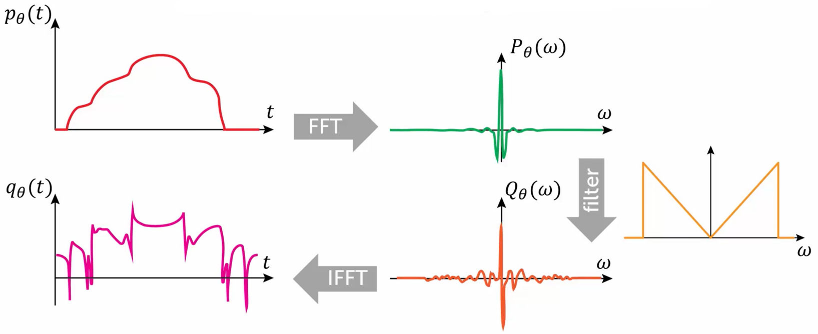
Filtered backprojection



Taken from corresponding video by the ASTRA toolbox team YouTube

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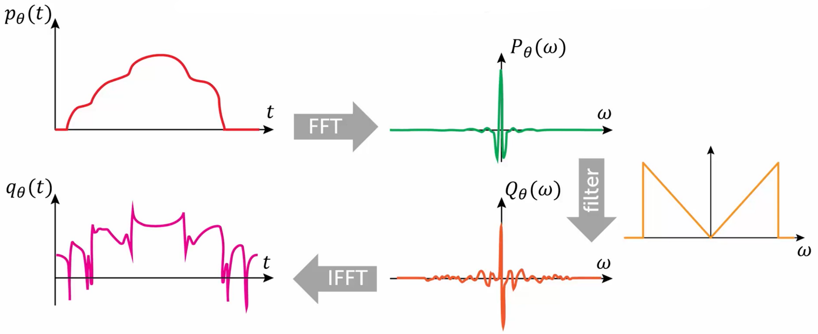
Filtered backprojection



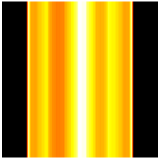
$$FBP\left[p(\theta,t)\right](x,y) := BP\left[q(\theta,t)\right](x,y) := \int q_\theta\left(x\cos\theta + y\sin\theta\right)d\theta$$

$$q_\theta(t) := \int \mathcal{F}[p_\theta](\omega)|\omega|e^{i2\pi\omega t}d\omega$$

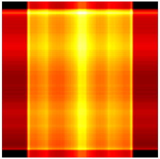
Taken from corresponding video by the ASTRA toolbox team YouTube

Filtered backprojection in action

It turns out that $FBP(\mathcal{R}f) = \mathcal{R}^*\mathcal{H}\mathcal{R}f = f$



(a) 1 angle

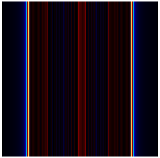


(b) 2 angles

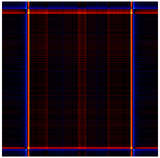
(c) 8 angles

(d) 64 angles

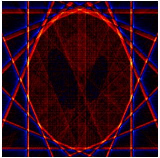
(e) 256 angles



(f) 1 angle



(g) 2 angles



(h) 8 angles

(i) 64 angles

(j) 256 angles

Just one more video by Samuli Siltanen: YouTube

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CT reconstruction methods

Analytical (or direct) methods a la filtered backprojection:

- ✓ efficient to implement and execute
- ! lack of flexibility for unconventional scanning set-ups
- ! severe artifacts for limited / sparse projection data
- ! hard to introduce a-priori knowledge

Algebraic and variational methods (iterative methods):

- ! higher computational cost
- ✓ highly flexible, arbitrary geometries
- ✓ less artifacts for limited / sparse projection data
- ✓ introduction of a-priori knowledge possible

Algebraic Reconstructions

Idea: Find $f \in \mathcal{C}$ with $p \approx \mathcal{R} f$ as

$$f = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \; \|\mathcal{R} f - p\|_2^2 \quad ,$$

for instance via projected gradient descent:

$$f^{k+1} = P_{\mathcal{C}} \left(f^k - \nu \mathcal{R}^* \left(\mathcal{R} f^k - p \right) \right)$$

Many variants of this exist such as ART, SART, SIRT, ...

Variational Reconstructions

$$f = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \; \mathcal{D}(\mathcal{R} f, p) + \lambda \mathcal{J}(f) \quad ,$$

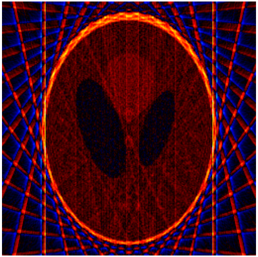
where $\mathcal{D}$ and $\mathcal{J}$ are derived from **probabilistic models** for data generation (*likelihood*) and typical images (*prior*), for instance

$$\mathcal{D}(\mathcal{R} f, g) := \left\| M^{-1/2} \left(\mathcal{R} f - p \right) \right\|_2^2, \qquad \mathcal{J}(f) := \| \nabla f \|_1$$

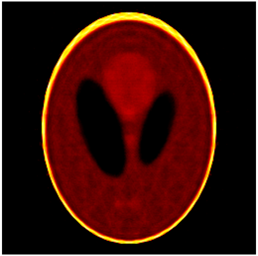
Solution via **iterative optimization schemes** such as proximal gradient descent, primal-dual hybrid gradient, alternating direction method of multipliers, ...

Iterative methods in action: 15 angles

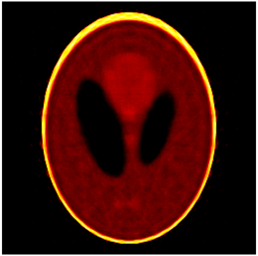
(a) true image



(b) FBP



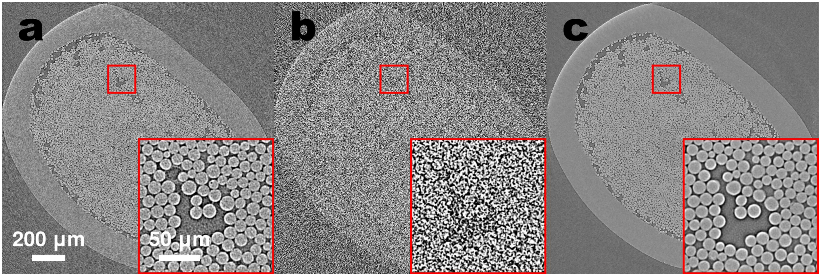
(c) ART



(d) SIRT

(e) TV regularization

Deep learning for low dose CT image reconstruction



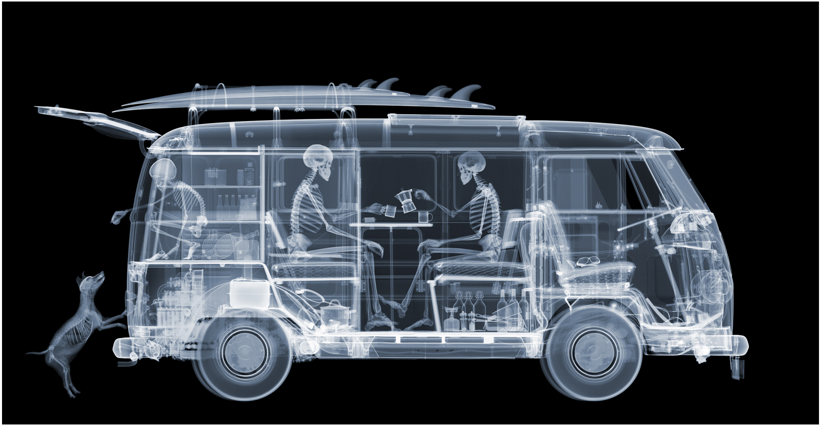
2560x2560 tomography images of fiber composite. Left: 1024 projections, middle/right: 128 projections

D. Pelt, J.A. Sethian, 2018. Mixed-scale dense network for image analysis, *PNAS 115 (2) 254-259.*

Some current developments

- Phase contrast X-ray imaging: Exploit phase shift in X-rays caused by material interaction to gain higher soft tissue contrast.
- Dynamic X-ray: Track fast dynamic processes in 3D (4D CT).
- Spectral CT: Use energy resolved detectors to improve analysis of complex materials and tissues.
- Scan adaptation: Make best possible use of given budget of radiation.
- Machine learning: Use deep learning to improve image reconstruction and analysis.

Break & questions time

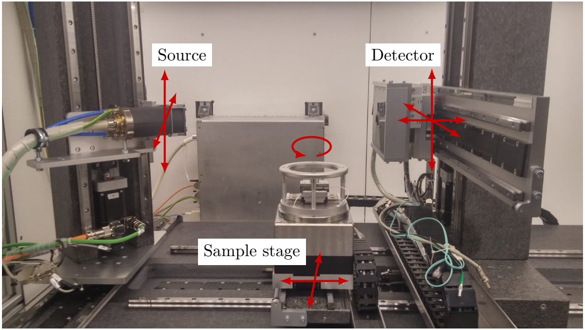


Nick Veasey, VW Camper Van , 2019

FleX-ray Lab at CWI

- custom-built, fully-automated, highly flexible
- linked to large-scale computing hardware
- **Aim: Proof-of-concept** experiments directly accessible to mathematicians and computer scientists.
- develop advanced computational techniques for 3D imaging

FleX-ray Lab at CWI



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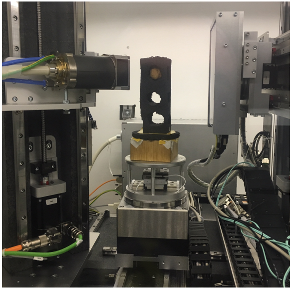
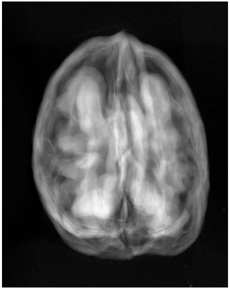
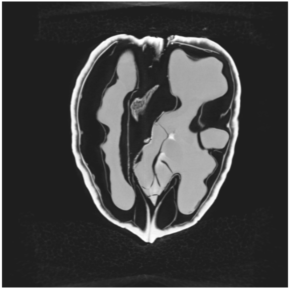
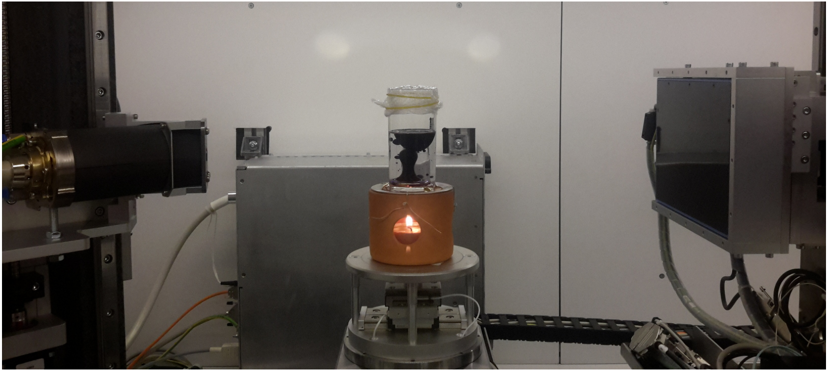
X-Ray Scan of static object

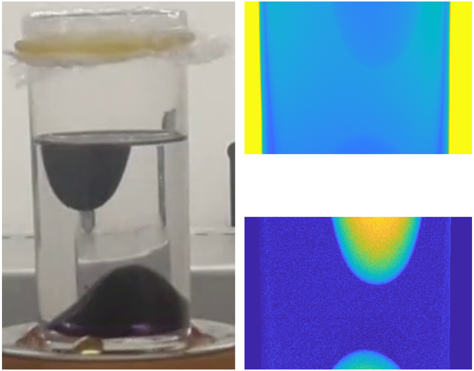




X-Ray Scan of Dynamic Object



- canonical example of temperature-driven **two-phase flow instability**
- 120 projections per rotation $\rightarrow$ each projection averaged over 3°
- 40ms exposure per projection $\rightarrow$ 4.8s per rotation

X-Ray Scan of Dynamic Object



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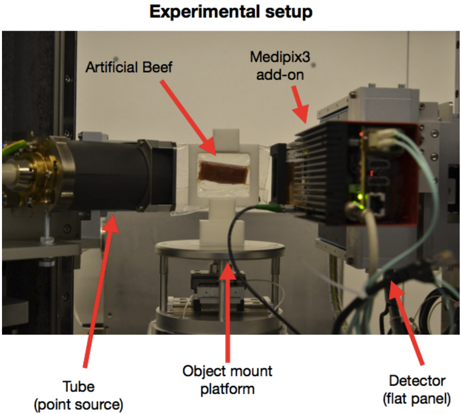
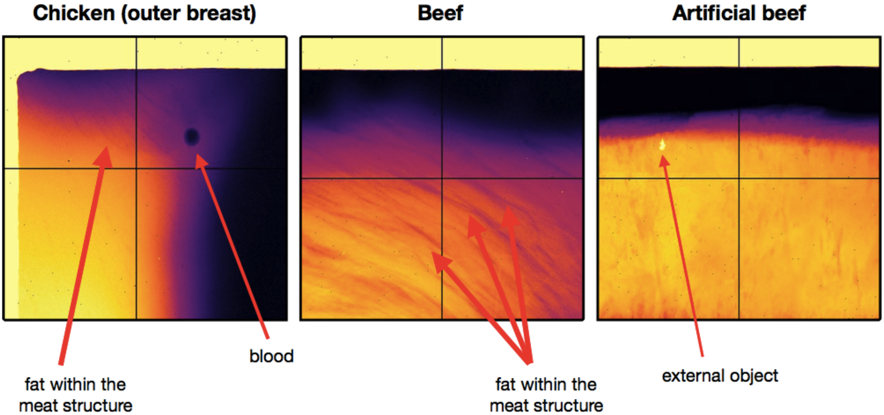
Foreign object detection with spectral CT



Samples included chicken breasts, thighs, skin, beef and pork meat and artificial meat

- template for many industry applications
- low quality data, high throughput

Foreign object detection with spectral CT



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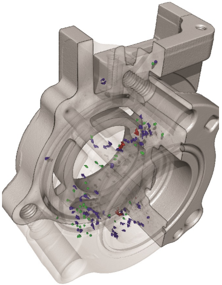
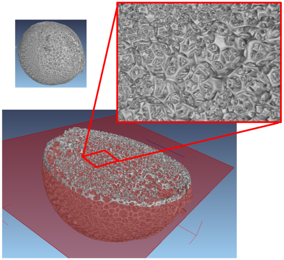
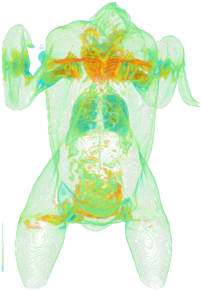
Example applications: Industry & security







Airport baggage screening YouTube

Example applications: Materials science, energy & biology

- Water within porous media YouTube
- Internal structure of Arundo donax YouTube
- Inside live flying insects – in 3D YouTube
- Movie of battery under load YouTube
- Metallic foam YouTube

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Further reading

- **T. M. Buzug, 2008.** Computed Tomography - From Photon Statistics to Modern Cone-Beam CT, *Springer-Verlag Berlin Heidelberg.*
- **G. T. Herman, 2009.** Fundamentals of Computerized Tomography Image Reconstruction from Projections, *Springer-Verlag London.*
- **F. Natterer, 2001.** The Mathematics of Computerized Tomography, *Society for Industrial and Applied Mathematics.*