Centrum Wiskunde & Informatica

Joint Tomographic Image Reconstruction and Motion Estimation

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SIAM Imaging Science
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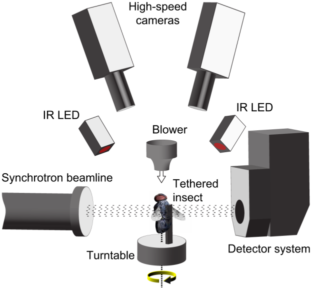
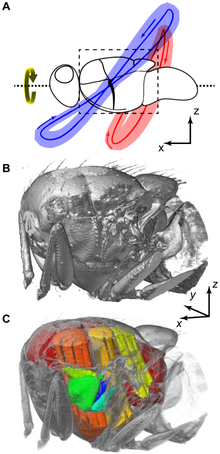
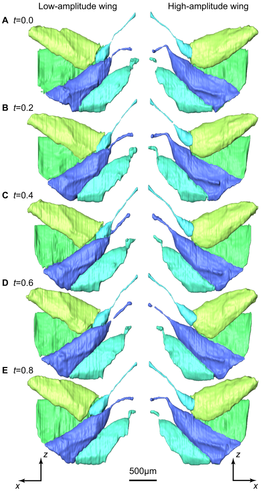
Example: In Vivo Time-Resolved Microtomography



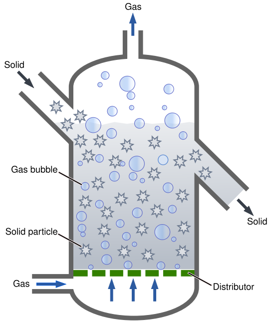


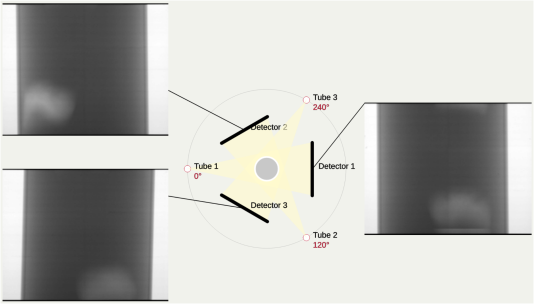


Walker, Schwyn, Mokso, Wicklein, Müller, 2014. In Vivo Time-Resolved Microtomography Reveals the Mechanics of the Blowfly Flight Motor, *PLoS Biol 12(3).*

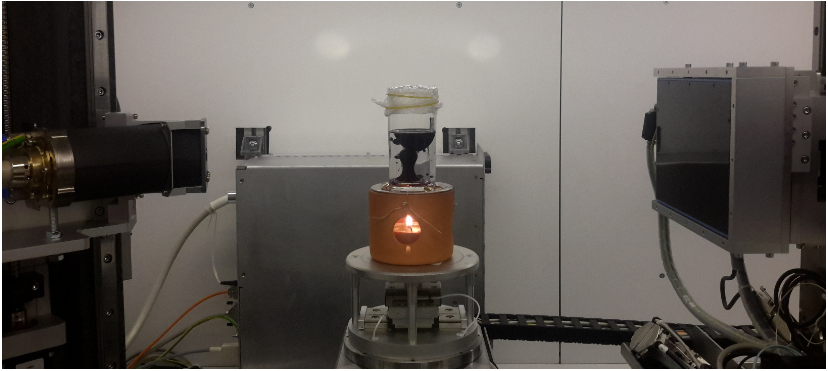
Example: Fluidized Bed Reactors

Collaboration with the Transport Phenomena group at TU Delft.



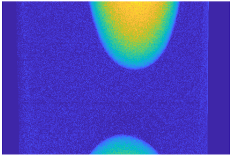


Own Experiments: Two-Phase Flow Instability



- canonical example of temperature-driven **two-phase flow instability**
- 120 projections per rotation $\rightarrow$ each projection averaged over 3°
- 40ms exposure per projection $\rightarrow$ 4.8s per rotation

Own Experiments: Two-Phase Flow Instability



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Overview Dynamic Imaging

Applications

- scientific, industrial and clinical
- vast range of dynamics (rigid motion, elastic deformation, fluid dynamics, crack formation, chemical kinetics, granular flows, ...)

Goals

- motion compensation
- gating
- full dynamic reconstruction (+ simultanous motion estimation?)
- parameter identification in dynamical systems

Challenges

- dynamics too fast for high quality frame-by-frame reconstruction (motion artefacts, noise, low angular res,...)
- mathematical modeling of dynamics
- computational image reconstruction

Overview Mathematical Setup and Challenges

static

$$f = Au^\dagger$$

dynamic

$$f(t) = C(t)Au^\dagger(t)$$

binned

$$\underbrace{\begin{bmatrix} f(t_i) \\ \vdots \\ f(t_j) \end{bmatrix}}_{f_{ij}} = \underbrace{\begin{bmatrix} C(t_i) \\ \vdots \\ C(t_j) \end{bmatrix}}_{C_{ij}} Au_{ij}$$

✓ static reconstruction of sufficient quality

! $C(t)$ reduces dimension of data

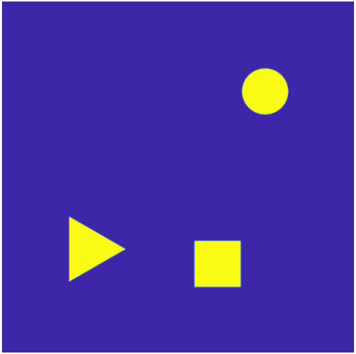
! trade-off:

- larger bins $\longrightarrow$ more motion-blur
- smaller bins $\longrightarrow$ more underdetermined

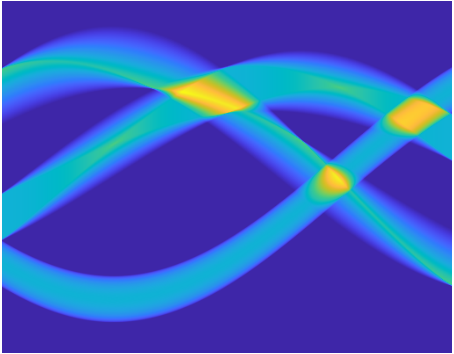
! sequential scanning is worst sub-sampling ($\neq$ "compressed sensing")

! high computational/memory demands

Illustration: Limited-View Artifacts



true image $u^\dagger$



sinogram $Au^\dagger$

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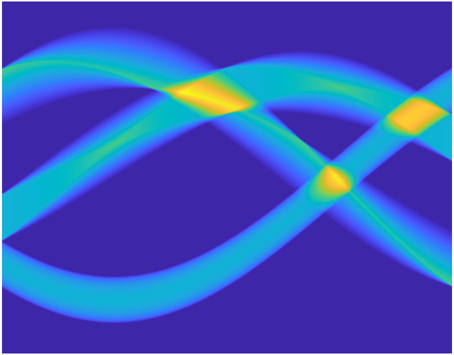
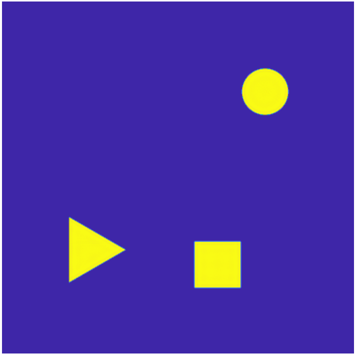


sinogram f

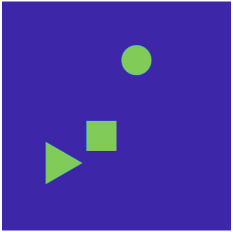


reconstruction $\hat{u}$

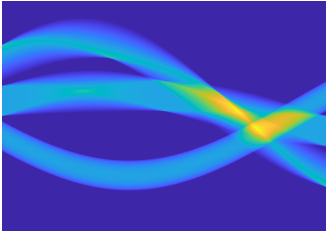
accelerated gradient descent to solve

$$\hat{u} = \underset{u \geq 0}{\text{argmin}} \; \|Au - f\|_2^2$$

Illustration: Motion Artifacts



$u^\dagger$



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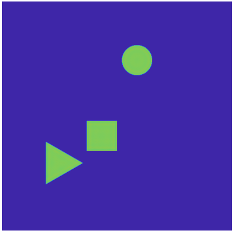

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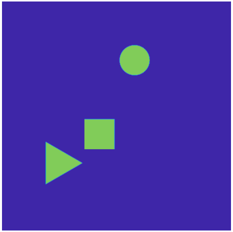

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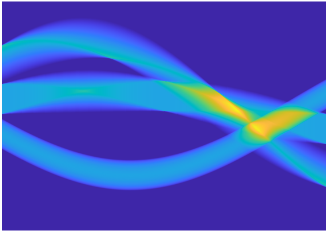

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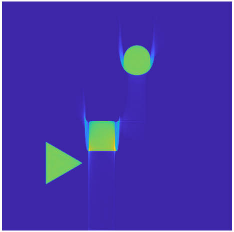

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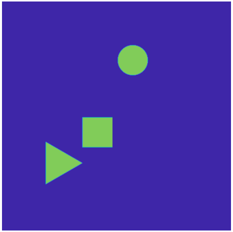

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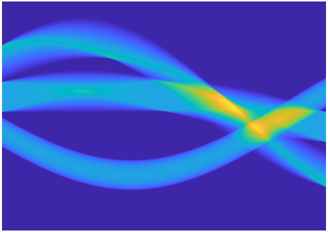

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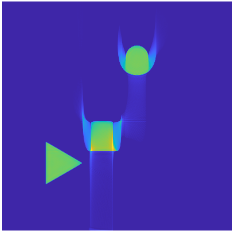

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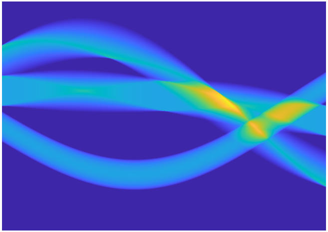

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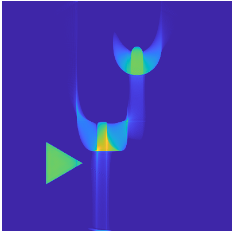

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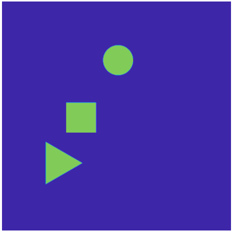

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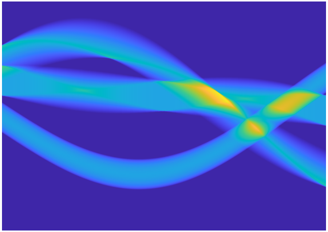

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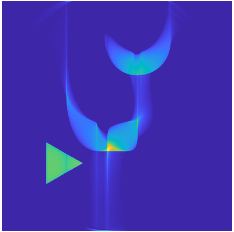

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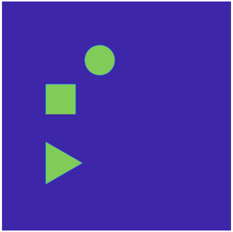

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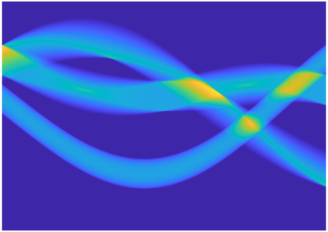

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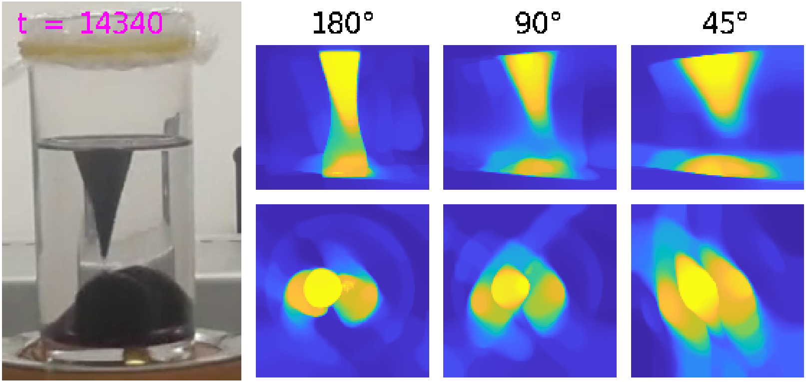
Lava Lamp: Frame-by-Frame Reconstruction



Spatio-Temporal Variational Approaches

variational forward filtering

$$\hat{u}_i = \underset{u \geqslant 0}{\operatorname{argmin}} \; \tfrac{1}{2} \left\| C_i A u - f_i \right\|_2^2 + \alpha \mathcal{J}(u, \hat{u}_{i-1})$$

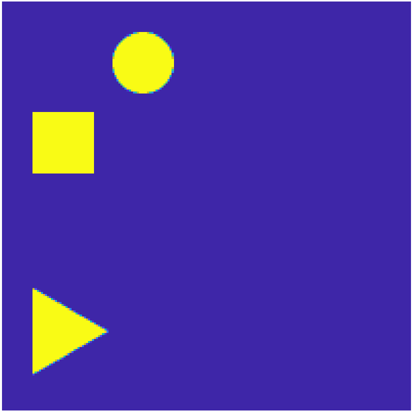
e.g., $\mathcal{J}(u, \hat{u}_{i-1}) = \left\| \nabla u \right\|_1 + \gamma \left\| u - \hat{u}_{i-1} \right\|_1$

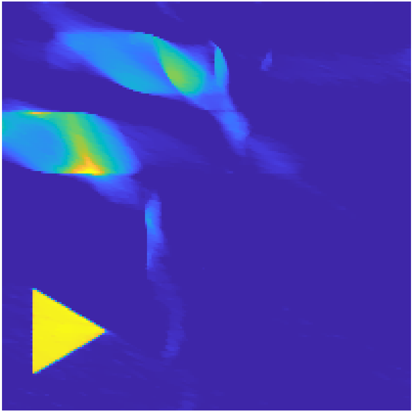
full spatio-temporal scheme

$$\hat{u} = \underset{u \geqslant 0}{\operatorname{argmin}} \; \tfrac{1}{2} \sum_i \left\| C_i A u_i - f_i \right\|_2^2 + \alpha \mathcal{J}(u)$$

e.g., $\mathcal{J}(u) = \sum_i \left\| \nabla u_i \right\|_1 + \gamma \left\| u_i - u_{i-1} \right\|_1$

Illustration: Total Variation in Space and Time





large γ

$$\hat{u} = \underset{u \geqslant 0}{\operatorname{argmin}} \; \frac{1}{2} \sum_i \| C_i A u_i - f_i \|_2^2 + \alpha \| \nabla u_i \|_1 + \gamma^2/2 \, \| u_i - u_{i-1} \|_2^2$$

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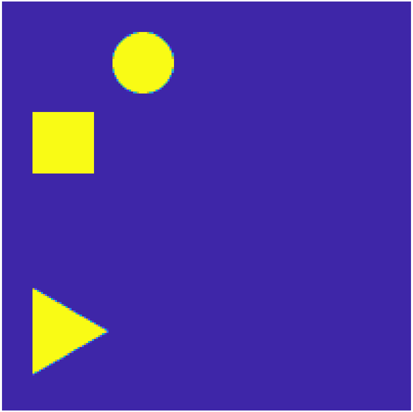

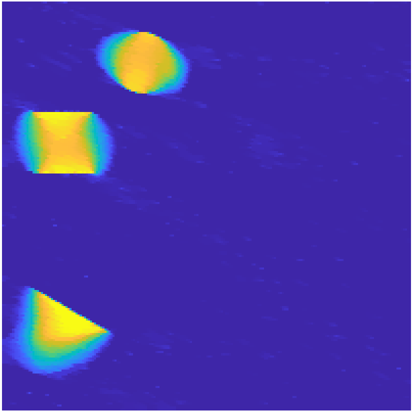

small γ

$$\hat{u} = \underset{u \geqslant 0}{\mathrm{argmin}} \ \frac{1}{2} \sum_i \| C_i A u_i - f_i \|_2^2 + \alpha \| \nabla u_i \|_1 + \gamma^2/2 \, \| u_i - u_{i-1} \|_2^2$$

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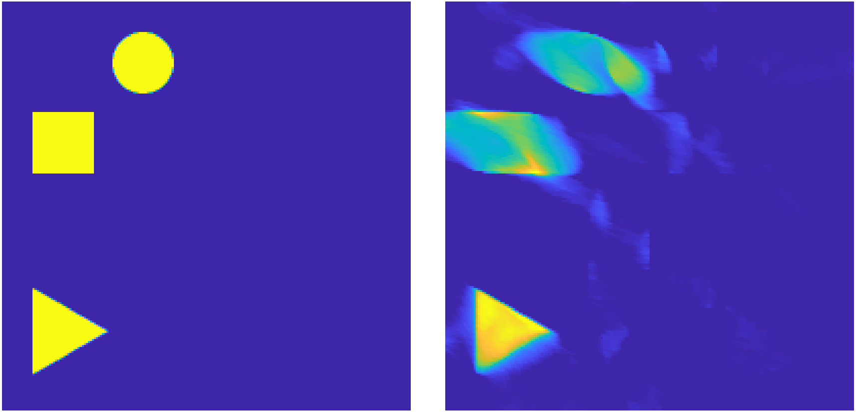

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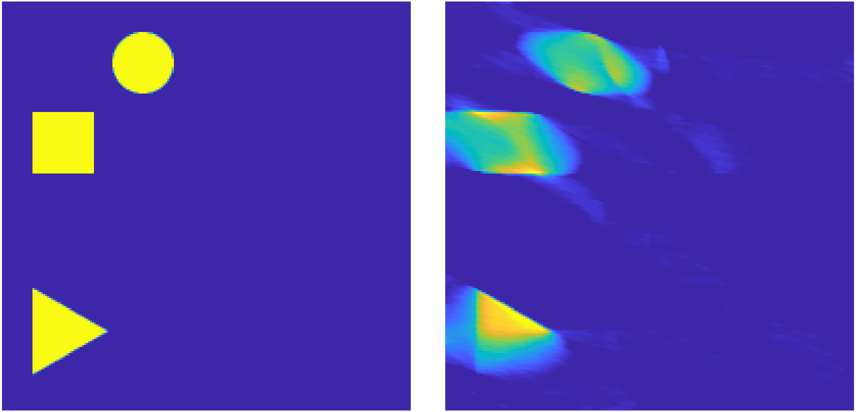

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Variational Space-Time Decomposition

Implicit spatio-temporal decomposition via **infimal convolutions**:

$$\hat{u} = \underset{u \geqslant 0, w}{\operatorname{argmin}} \ \frac{1}{2} \sum_i \| C_i A u_i - f_i \|_2^2 + \alpha TV_{\beta_1}(u - w) + \gamma TV_{\beta_2}(w)$$

Holler, Kunisch, 2014. On infimal convolution of TV type functionals and applications to video and image reconstruction, *SIAM IS.*

Schloegl, Holler, Schwarzl, Bredies, Stollberger, 2017. Infimal Convolution of Total Generalized Variation Functionals for Dynamic MRI, *MAGN.*

Explicit decomposition of Casorati matrix via **low-rank + sparsity** models:

$$\hat{u} = \underset{U_l, U_S}{\operatorname{argmin}} \ \frac{1}{2} \| A(U_L + U_S) - F \|_2^2 + \alpha_L \| L U_L \|_* + \alpha_S \| S U_S \|_1$$

Ravishankar, Ye, Fessler, 2019. Image Reconstruction: From Sparsity to Data-adaptive Methods and Machine Learning, *arXiv:1904.02816.*

introduce v as image-like hidden (*latent*) variable to construct

$$\mathcal{J}(u) := \min_{v} \; \mathcal{M}(u, v) + \mathcal{H}(v)$$

Variational Latent Variable Models

introduce v as image-like hidden (*latent*) variable to construct

$$\mathcal{J}(u) := \min_{v} \; \mathcal{M}(u, v) + \mathcal{H}(v)$$

joint image and latent variable reconstruction:

$$(\hat{u}, \hat{v}) = \underset{u,v}{\operatorname{argmin}} \; \frac{1}{2} \sum_{i=1} \| C_i A u_i - f_i \|_2^2 + \alpha \mathcal{R}(u_i) + \beta \mathcal{H}(v_i) + \gamma \mathcal{M}(u, v)$$

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Now: v motion field, $\mathcal{M}(u, v)$ encodes PDE model of image dynamics

Burger, Dirks, Schönlieb, 2018. A Variational Model for Joint Motion Estimation and Image Reconstruction, *SIAM IS.*

Excursus Motion Estimation

Continuous Derivation

Assume intensity $u(x,t)$ constant along $x(t)$ with $\frac{dx}{dt} = \tilde{v}(x,t)$ (velocity):

$$0 = \frac{d}{dt} u(x(t),t) = \frac{\partial u}{\partial t} + \sum_{k=1}^{d} \frac{\partial u}{\partial x_k} \frac{\partial x_k}{\partial t} = \partial_t u + (\nabla u) \cdot \tilde{v}$$

$\rightsquigarrow$ continuity PDE.

Given $u_1(x) := u(x,t_1)$, $u_2(x) := u(x,t_2)$
net displacement/motion field

$$v(x) = \int_{t_1}^{t_2} q(t) dt, \qquad \text{where } q(t) \text{ solves } \frac{d}{dt} q(t) = \tilde{v}(q(t),t), \quad q(t_1) = x$$

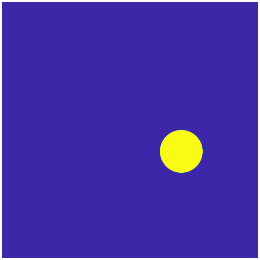
optical flow equation:

$$u_2(x + v(x)) = u_1(x)$$

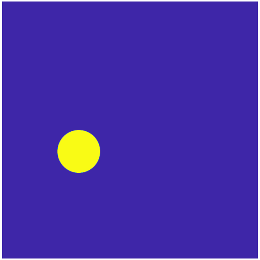
! underdetermined !

Illustration Underdeterminedness

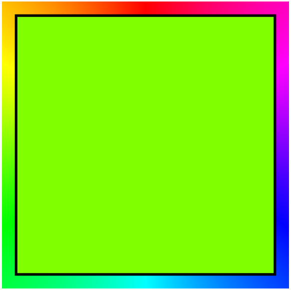
$$u_2(x + v(x)) = u_1(x)$$



u_1



u_2



v_1

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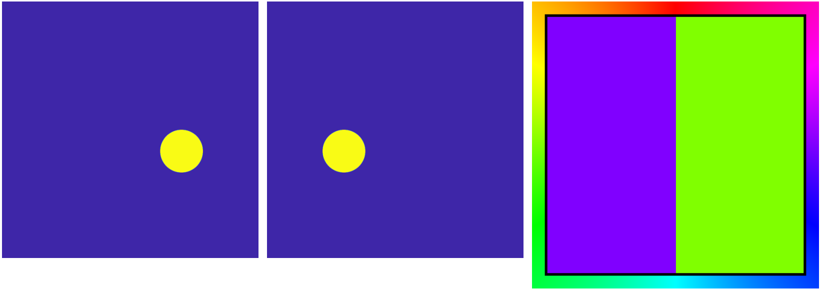
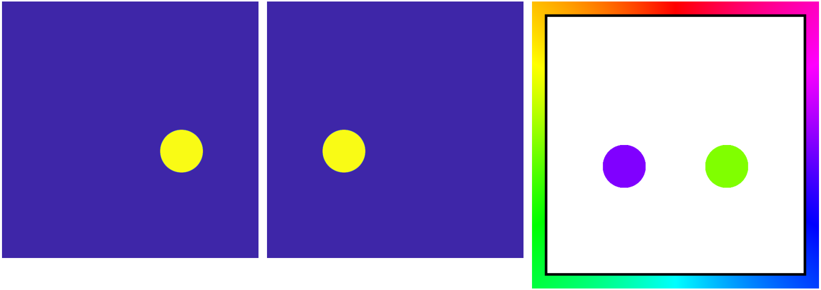
$$u_2(x + v(x)) = u_1(x)$$



u_1 u_2 v_2

$$u_2(x + v(x)) = u_1(x)$$



u_1 u_2 v_3

Illustration Underdeterminedness

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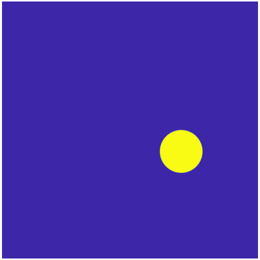

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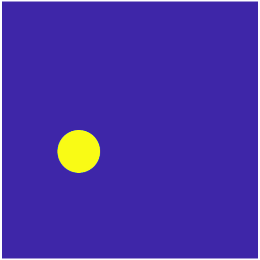
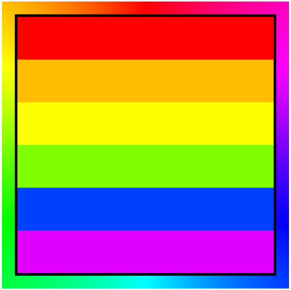


u_2



v_4

Variational Motion Estimation

regularized motion estimation

$$\min_v \mathcal{H}(v) \quad \textit{such that} \quad u_2(x + v(x)) = u_1(x)$$

$$\min_v \frac{1}{p} \| u_2(x + v(x)) - u_1(x) \|_p^p + \beta \mathcal{H}(v)$$

! terribly non-convex !

Variational Motion Estimation

regularized motion estimation

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! terribly non-convex !

small displacements around $\bar{v}(x)$:

$$u_2(x + v(x)) \approx \bar{u}_2(x) + (\nabla \bar{u}_2(x)) \cdot (v(x) - \bar{v}(x))$$

$$\bar{u}_2(x) := u_2(x + \bar{v}(x)) =: \mathcal{W}_{\bar{v}} u_2 \qquad \textbf{(warping)}$$

$$\min_v \frac{1}{p} \| \bar{u}_2 - (\nabla \bar{u}_2) \cdot \bar{v} - u_1 + (\nabla \bar{u}_2) \cdot v \|_p^p + \beta \mathcal{H}(v)$$

for $\bar{v} = 0$ we get to backward-discretized PDE back:

$$\min_v \frac{1}{p} \| u_2 - u_1 + (\nabla u_2) \cdot v \|_p^p + \beta \mathcal{H}(v)$$

Computational Solution I

linearized problems:

$$\min_{v} \; \frac{1}{p} \|a \cdot v - f\|_p^p + \beta \mathcal{H}(v)$$

convex but typically non-smooth, e.g. $p = 1$, $\mathcal{H}(v) = TV(v)$

Primal-dual hybrid gradient: too slow convergence in 3D

Alternating directions method of multipliers (ADMM):

- ! more difficult to parameterize
- ! badly conditioned, large-scale least-squares problems
- ! choice of iterative solver crucial, here **AMG-CG**

Chambolle, Pock, 2016. An introduction to continuous optimization for imaging, *Acta Numerica.*

Computational Solution II

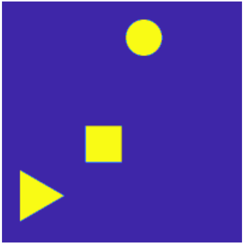
$$\min_{v} \; \frac{1}{p} \| u_2(x + v(x)) - u_1(x) \|_p^p + \beta \mathcal{H}(v)$$

coarse-to-fine scheme:

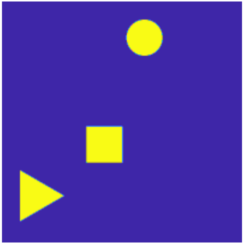
- successive smoothing & down-sampling of u_2, u_1
- successive solution of linearized problems on each level
- upsampling of solution and aux variables to finer level
- optional: add line search

Brox, Bruhn, Papenberg, Weickert, 2004. High Accuracy Optical Flow Estimation Based on a Theory for Warping, *ECCV 2004.*

Illustration Small Displacements



u_1



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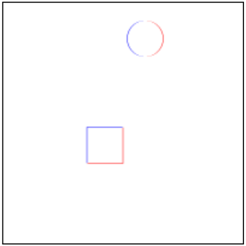
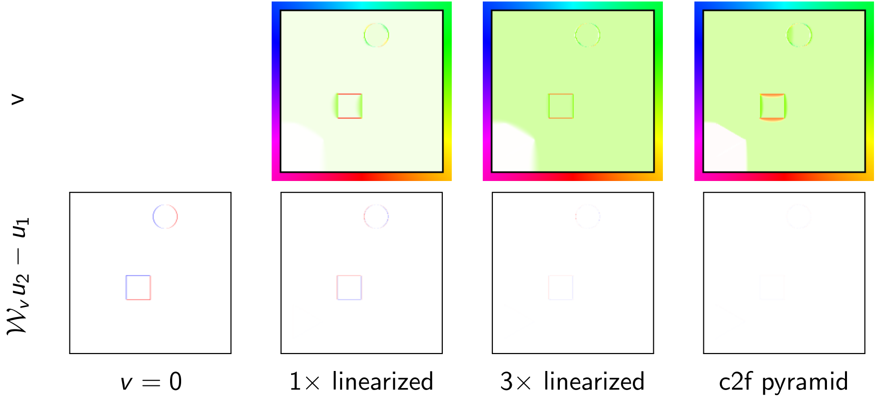


$u_2 - u_1$

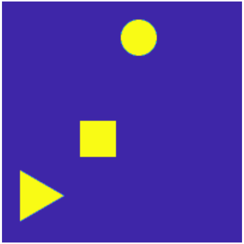
Illustration Small Displacements



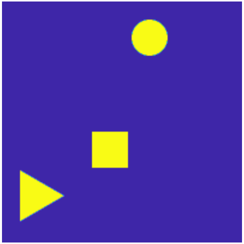
$\|\mathcal{W}_v u_2 - u_1\|$:

5.7	3.3	1.1	0.2

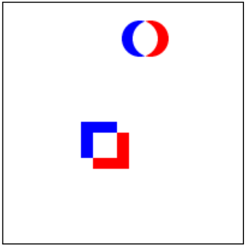
Illustration Medium Displacements



u_1

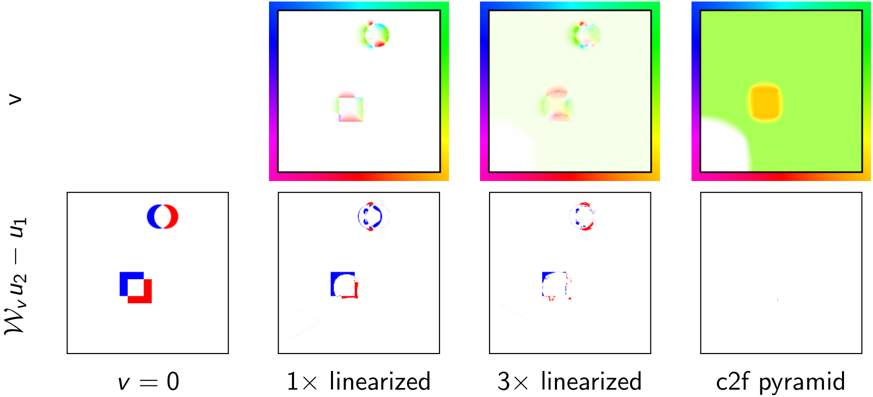


u_2



$u_2 - u_1$

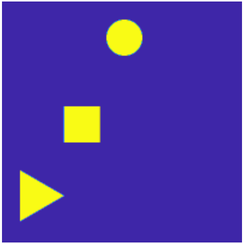
Illustration Medium Displacements



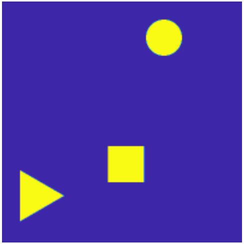
$\|\mathcal{W}_v u_2 - u_1\|$:

38.3	23.8	15.3	0.6

Illustration Large Displacements



u_1



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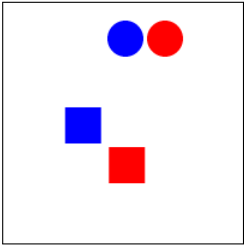
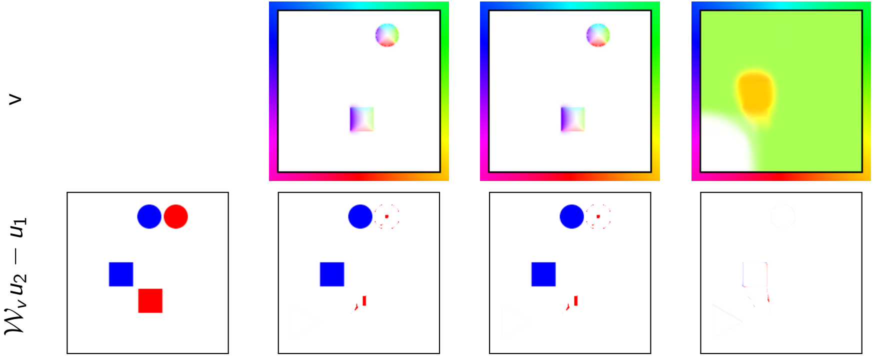


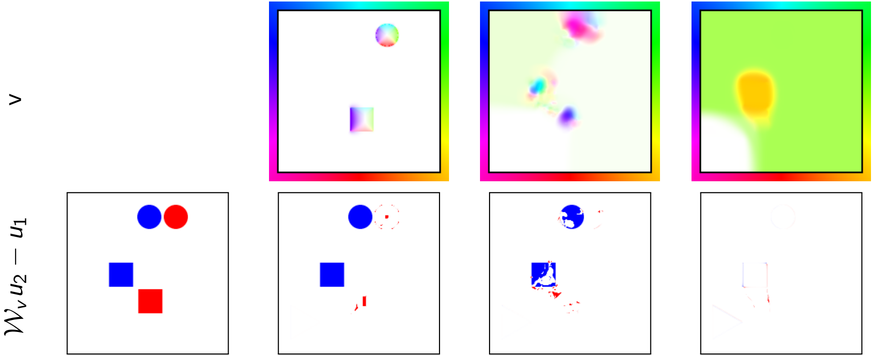
$u_2 - u_1$

Illustration Large Displacements



	$v = 0$	$1\times$ linearized	$3\times$ linearized	c2f pyramid
$\|\mathcal{W}_v u_2 - u_1\|$:	56.3	40.9	40.9	2.7

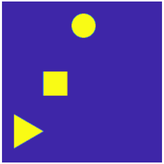
Illustration Pyramid Schemes



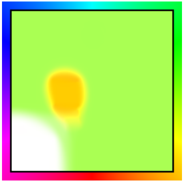
	$v = 0$	1 level, 3 lin	5 level, 3 lin	23 level, 3 lin
$\|\mathcal{W}_v u_2 - u_1\|$:	56.3	40.9	33.7	2.7

Illustration Symmetric OF

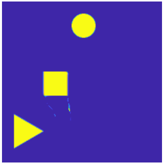
$$v^{fwd} = \underset{v}{\operatorname{argmin}} \ \frac{1}{2} \left\| u_2(x+v) - u_1(x) \right\|_2^2 + \beta \mathcal{H}(v)$$



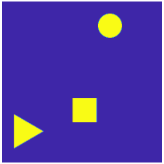
u_1



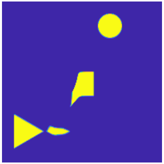
v^{fwd}



$u_2(x + v^{fwd})$



u_2

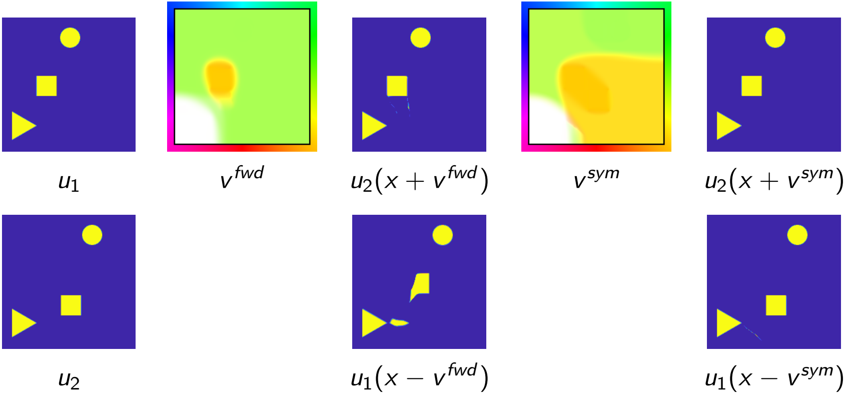


$u_1(x - v^{fwd})$

Illustration Symmetric OF

$$v^{fwd} = \underset{v}{\operatorname{argmin}} \ \frac{1}{2} \left\| u_2(x+v) - u_1(x) \right\|_2^2 + \beta \mathcal{H}(v)$$

$$v^{sym} = \underset{v}{\operatorname{argmin}} \ \frac{1}{4} \left\| u_2(x+v) - u_1(x) \right\|_2^2 + \frac{1}{4} \left\| u_1(x-v) - u_2(x) \right\|_2^2 + \beta \mathcal{H}(v)$$



Back to
Image Reconstruction

Joint Image Reconstruction and Motion Estimation

$$\min_{u,v} \left(\mathcal{E}(u,v) = \frac{1}{2} \sum_{i=1} \| C_i A u_i - f_i \|_2^2 + \alpha \mathcal{R}(u_i) + \beta \mathcal{H}(v_i) + \gamma \mathcal{M}(u,v) \right)$$

$$\mathcal{M}_p^l \ (u,v) := \sum_i \frac{1}{p} \| u_{i+1} - u_i + (\nabla u_{i+1}) \cdot v_i \|_p^p$$

$$\mathcal{M}_p^{nl} \ (u,v) := \sum_i \frac{1}{p} \| \mathcal{W}_{v_i} u_{i+1} - u_i \|_p^p \qquad (\mathcal{M}_p^{nls}(u,v) \text{ symmetric})$$

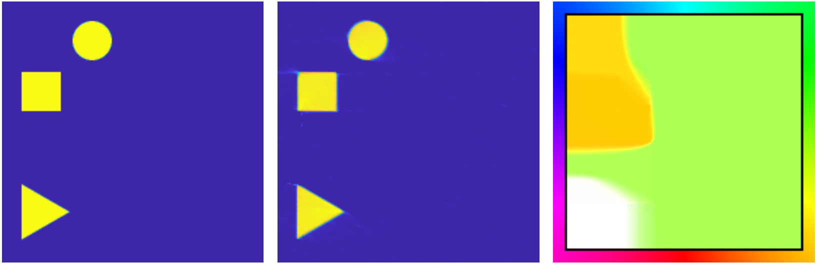
- $\mathcal{R}(u_i)$, $\mathcal{H}(v_i)$ typically **convex but non-smooth**, e.g., TV
- $\mathcal{E}(u,v)$ convex in u
- $\mathcal{M}_p^l(u,v)$ convex in $v \Longrightarrow \mathcal{E}(u,v)$ **bi-convex** in (u,v)

alternating optimization:

$$u^{k+1} = \operatorname*{argmin}_u \ \mathcal{E}(u, v^k) \qquad \text{image estimation, convex, non-smooth}$$

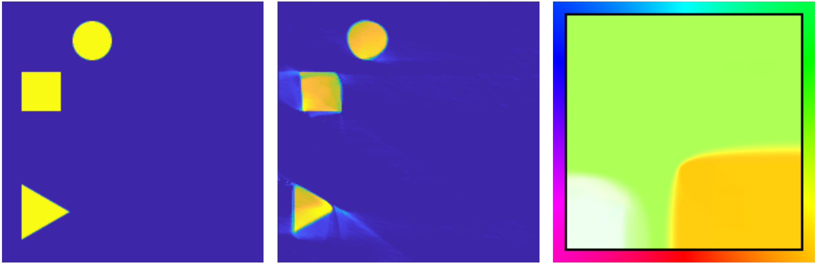
$$v^{k+1} = \operatorname*{argmin}_v \ \mathcal{E}(u^{k+1}, v) \qquad \text{motion estimation, (non-)convex, non-smooth}$$

Simulation Results: Motion Oracle



$$\min_{u} \ \frac{1}{2} \sum_{i=1} \| C_i A u_i - f_i \|_2^2 + \alpha TV(u_i) + \beta TV(v_i)$$
$$+ \gamma^2/2 \left(\left\| \mathcal{W}_{v_i^\dagger} u_{i+1} - u_i \right\|_2^2 + \left\| \mathcal{W}_{-v_i^\dagger} u_i - u_{i+1} \right\|_2^2 \right)$$

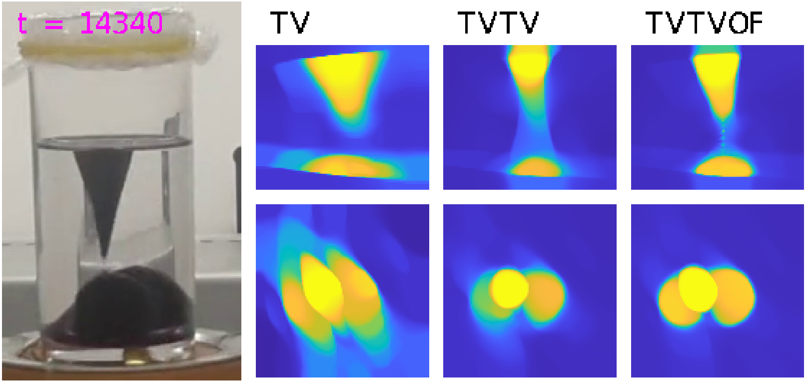
Simulation Results: Joint Estimation



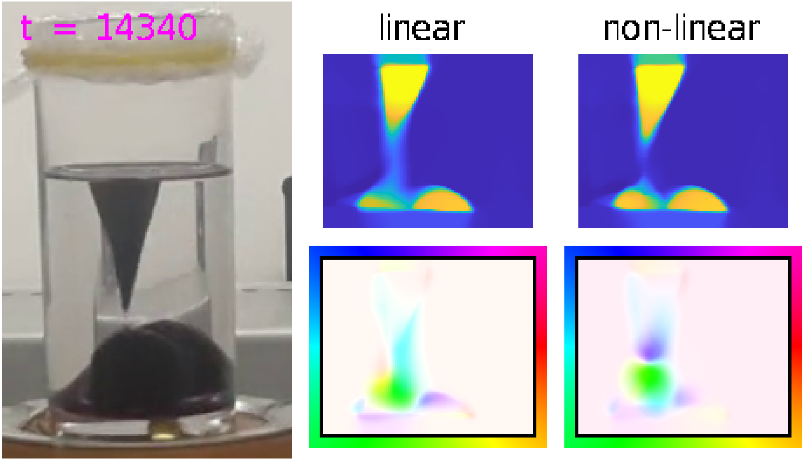
$$\min_{u,v} \ \frac{1}{2} \sum_{i=1} \| C_i A u_i - f_i \|_2^2 + \alpha TV(u_i) + \beta TV(v_i)$$

$$+ \gamma^2/2 \left(\| \mathcal{W}_{v_i} u_{i+1} - u_i \|_2^2 + \| \mathcal{W}_{-v_i} u_i - u_{i+1} \|_2^2 \right)$$

Lava Lamp Results



Lava Lamp: Image and Motion Estimation



Summary

- dynamic imaging is **hard in both theory and practice**
- **trade-off** between spatial and temporal resolution
- sequential scanning is the worst
- spatio-temporal regularization via **hidden variable models**
- joint image reconstruction and motion estimation
- heavy numerical optimization
- motion estimation is tough

- **L, Coban, Grass, Lagerwerf, Der Sarkissian, Palenstijn, Batenburg, 2020.** Dynamic Tomography of Rapid Deformations with Sequential Scanning, *in preparation.*
- **L, Huynh, Betcke, Zhang, Beard, Cox, Arridge, 2018.** Enhancing Compressed Sensing Photoacoustic Tomography by Simultaneous Motion Estimation, *SIAM Imaging Sciences 11 (4).*

Thank you for your attention!

- **L, Coban, Grass, Lagerwerf, Der Sarkissian, Palenstijn, Batenburg, 2020.** Dynamic Tomography of Rapid Deformations with Sequential Scanning, *in preparation.*
- **L, Huynh, Betcke, Zhang, Beard, Cox, Arridge, 2018.** Enhancing Compressed Sensing Photoacoustic Tomography by Simultaneous Motion Estimation, *SIAM Imaging Sciences 11 (4).*