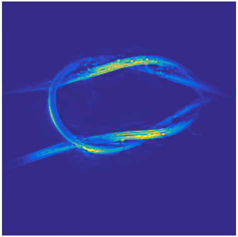
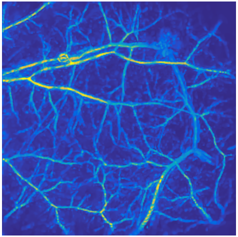
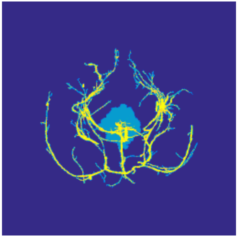
4D PAT based on Sparse Variational Methods







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joint with:
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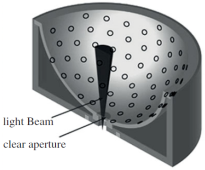
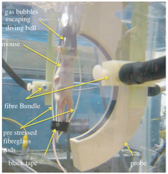
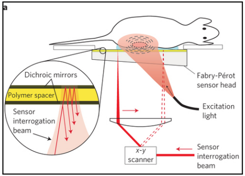
Marta Betcke, Ben Cox,
Nam Huynh & Edward Zhang

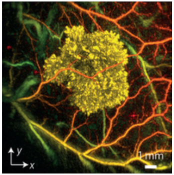
Orléans
March 09, 2016.

Photoacoustic Sensing Systems



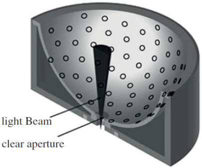


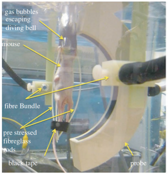


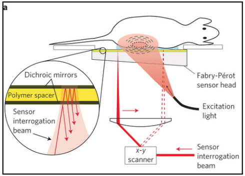


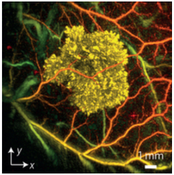
from: **Beard, 2011**, *Interface Focus*; **Jathoul et al., 2015**, *Nature Photonics*

Photoacoustic Sensing Systems



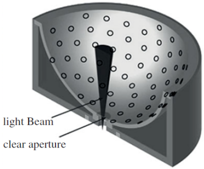


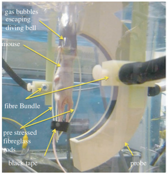


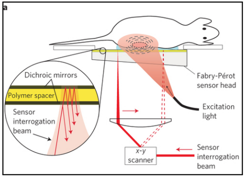


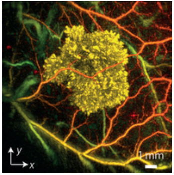
from: **Beard, 2011**, *Interface Focus*; **Jathoul et al., 2015**, *Nature Photonics*

- High res 3D PA images require sampling acoustic waves with a frequency content in the tens of MHz over cm scale apertures.
- Nyquist criterion results in tens of µm scale sampling intervals $\Longrightarrow$ several thousand detection points.
- Sequential scanning currently takes several minutes.
- Parallelized schemes (arrays) become prohibitively expensive.
- Crucial limitation for clinical, spectral and dynamical PAT (4D PAT).









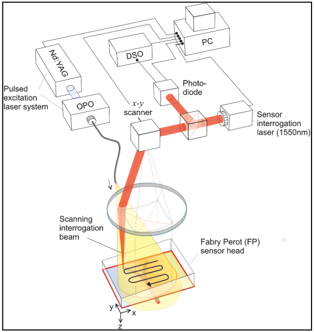
from: **Beard, 2011**, *Interface Focus*; **Jathoul et al., 2015**, *Nature Photonics*

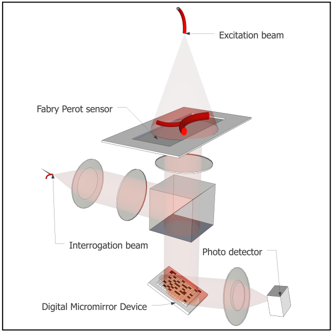
Key observation and idea:

- Nyquist is too conservative as only band-limitlessness is assumed.
- Typical targets have additional structure, e.g., low spatial complexity (sparsity).
- Regularly sampled data is highly redundant.
- Non-redundant part could be sensed faster.
- Accelerated acquisition without significant loss of image quality.

Established as compressed sensing, successful in similar modalities.

Novel Fabry-Pérot-Based Sensing Systems

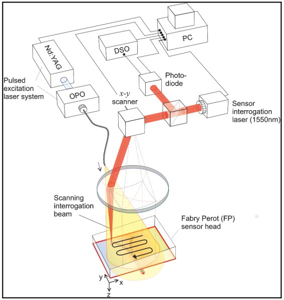




- Single-point sub-sampling (structured or random).
- Patterned interrogation by micromirror array, similar to "single-pixel" Rice camera.
- Multi-beam scanning + sub-sampling.

Applicable to other sequential scanning schemes, we focused on Fabry Pérot interferometer.
See **Huynh et al., 2014, 2015, 2016** for technical details.

PA Image Reconstruction from Sub-Sampled Data



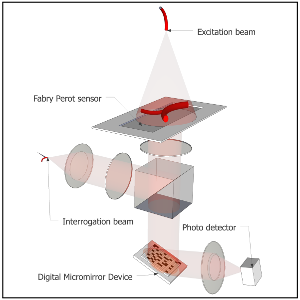


Image model: $\boxed{f_i^c = C_i f_i = C_i(Ap_i + \varepsilon_i)}$ for each frame i.

Image reconstruction:

- $f_i^c \longrightarrow f_i$, $f_i \longrightarrow p_i$ by standard method, frame-by-frame.
- $f_i^c \longrightarrow p_i$: standard or new method, frame-by-frame.
- $F^c \longrightarrow F$, $f_i \longrightarrow p_i$ by standard method, frame-by-frame.
- $F^c \longrightarrow P$: Full spatio-temporal method.

Standard Reconstruction & Numerical Wave Propagation

Analytic methods, e.g. eigenfunction expansion and closed-form filtered-backprojection, are too restrictive for us.

Time Reversal (TR):

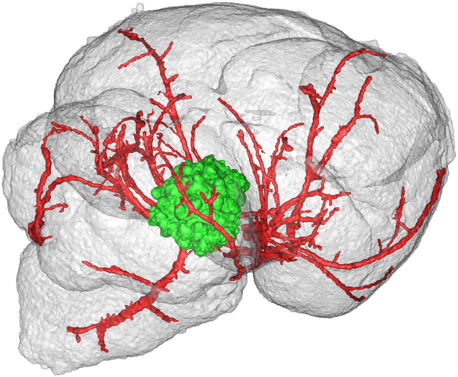
- "Least restrictive PAT reconstruction"
- Sending the recorded waves "back" into volume.
- Requires a numerical model for acoustic wave propagation.

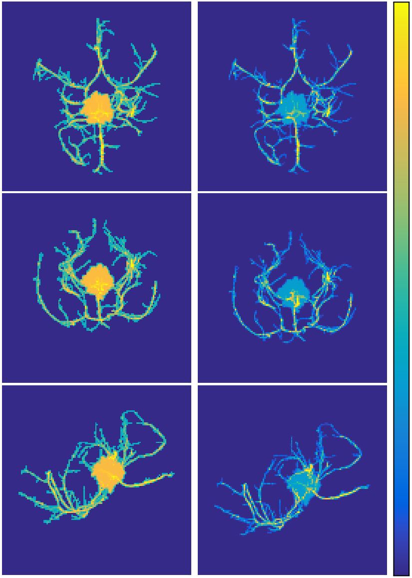
k-Wave(*) implements a *k*-space pseudospectral method to solve the underlying system of first order conservation laws:

- Compute spatial derivatives in Fourier space: 3D FFTs.
- Modify finite temporal differences by *k*-space operator and use staggered grids for accuracy and robustness.
- Perfectly matched layer to simulate free-space propagation.
- Parallel/GPU computing leads to massive speed-ups.

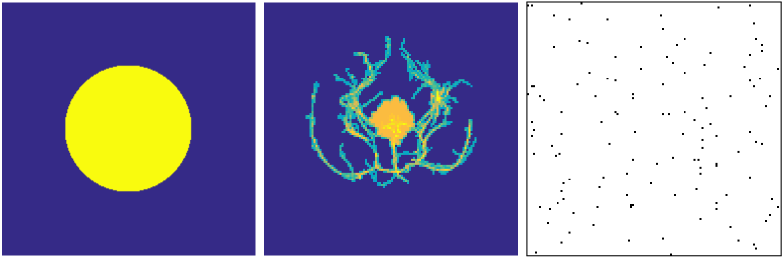
(*)**B. Treeby and B. Cox, 2010**. *k-Wave: MATLAB toolbox for the simulation and reconstruction of photoacoustic wave fields*, *Journal of Biomedical Optics*.

A Realistic Numerical Phantom

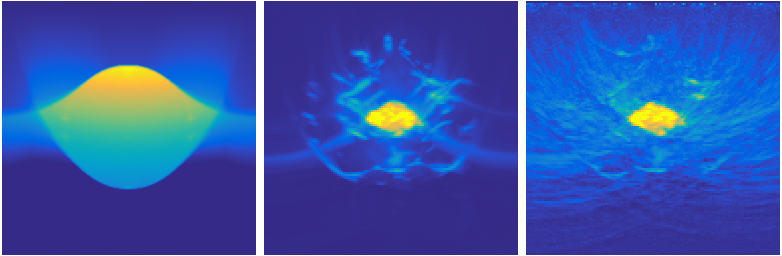




Time Reversal for Sub-Sampled Data



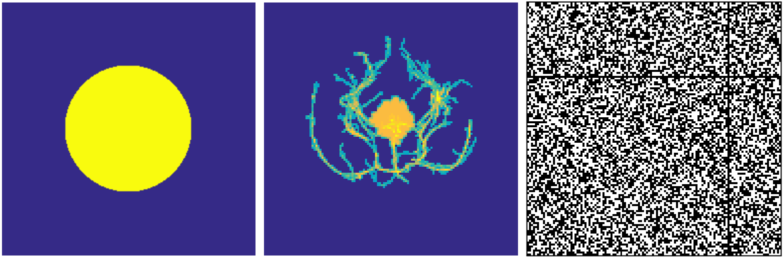
(a) IC, $n = 256^3$ (b) high con., IC, $n = 128^3$ (c) sub-sampling, 128x



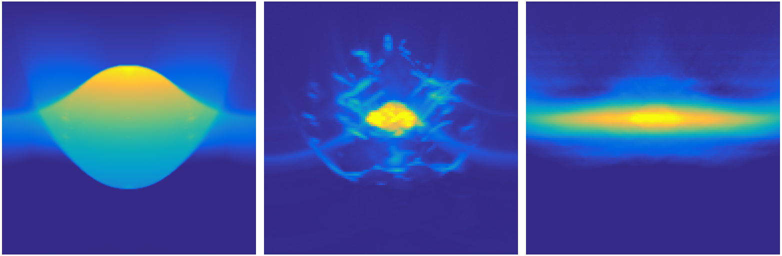
(d) TR 1 (e) TR 2 (f) TR 2, sub-sampled

sensor on top; inverse crime data sampled at Nyquist; max intensity proj., side view

Time Reversal for Sub-Sampled Data II



(a) IC, $n = 256^3$ (b) high con., IC, $n = 128^3$ (c) sub-sampling, 1/128



(d) TR 1 (e) TR 2 (f) TR 2, sub-sampled

sensor on top; inverse crime data sampled at Nyquist; max intensity proj., side view

Solving variational regularization problems

$$\hat{p} = \underset{p \geqslant 0}{\text{argmin}} \left\{ \tfrac{1}{2} \| CAp - f^c \|_2^2 + \lambda \mathcal{J}(p) \right\}$$

iteratively by first-order methods requires implementation of A and A^*.

k-Wave yields a discrete representation A_κ. For A^*, one can

1) adjoint k-Wave iteration to obtain $(A_\kappa)^*$ (algebraic adjoint):

- ✓ high numerical accuracy.
- ! tedious derivation, specific for k-Wave, limited insights.

Huang, Wang, Nie, Wang, Anastasio, 2013. *IEEE Trans Med Imaging*

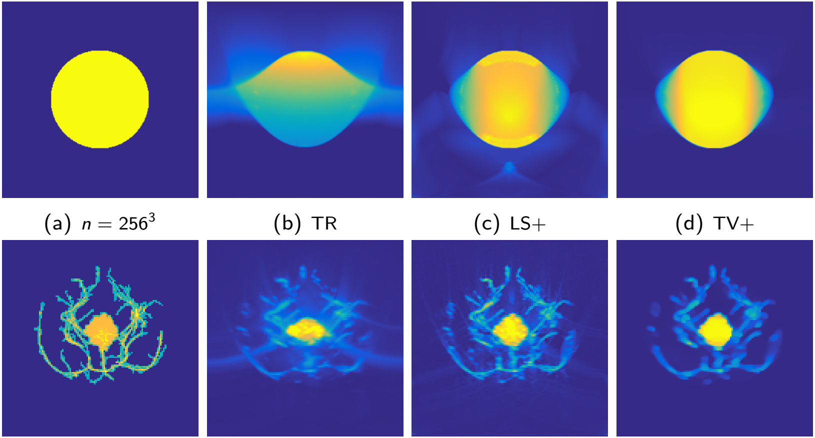
2) derive analytical adjoint and discretize it, e.g., $(A^*)_\kappa$.

- ✓ good numerical accuracy.
- ✓ simple proof, theoretical insights, generalizes to various numerical schemes.

Arridge, Betcke, Cox, L, Treeby, 2015. *On the Adjoint Operator in Photoacoustic Tomography*, (*submitted, arXiv:1602.02027*).

Comparison for Full Data

$$\hat{p} = \underset{p \geqslant 0}{\operatorname{argmin}} \left\{ \frac{1}{2} \| Ap - f \|_2^2 + \lambda \mathcal{J}(p) \right\}$$



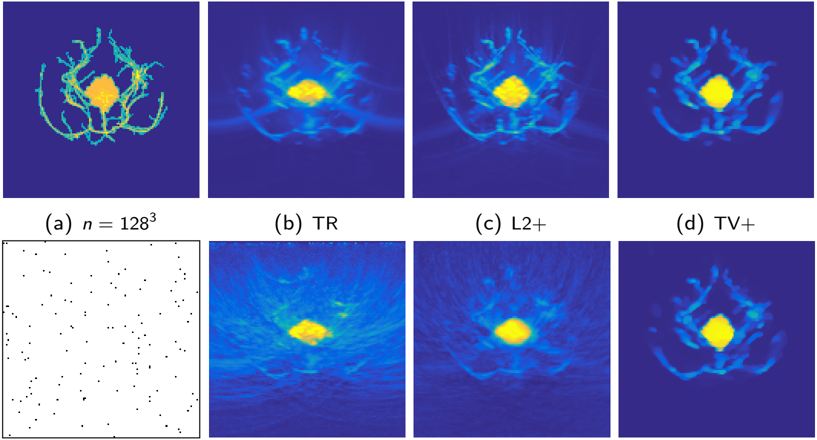
(a) $n = 256^3$ (b) TR (c) LS+ (d) TV+

(e) $n = 128^3$ (f) TR (g) LS+ (h) TV+

sensor on top; inverse crime data sampled at Nyquist; max intensity proj., side view

Sub Sampled Data, Best Case Scenario

$$\hat{p} = \underset{p \geqslant 0}{\operatorname{argmin}} \left\{ \frac{1}{2} \| CAp - f^c \|_2^2 + \lambda \mathcal{J}(p) \right\}$$



(a) $n = 128^3$ (b) TR (c) L2+ (d) TV+

(e) SubSam, 128x (f) TR (g) L2+ (h) TV+

sensor on top; inverse crime data sampled at Nyquist; max intensity proj., side view

Variational approaches,

$$\hat{p} = \underset{p}{\operatorname{argmin}} \left\{ \tfrac{1}{2} \| CAp - f^c \|_2^2 + \lambda \mathcal{J}(p) \right\},$$

suffer from systematic bias $\rightsquigarrow$ problem for quantitative use!
(e.g., contrast loss for TV).

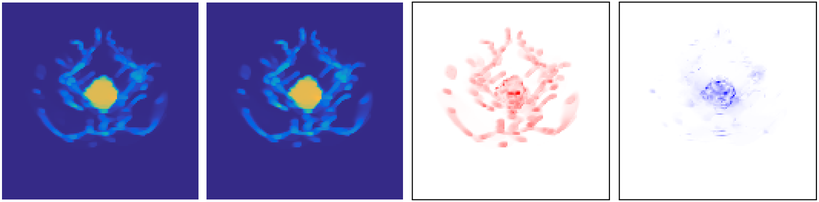
$\Longrightarrow$ Iterative enhancement trough Bregman iterations:

$$p^{k+1} = \underset{p}{\operatorname{argmin}} \left\{ \frac{1}{2} \| CA\, p - (f^c + b^k) \|_2^2 + \lambda \mathcal{J}(p) \right\}$$
$$b^{k+1} = b^k + \left(f^c - CA\, p^{k+1} \right)$$

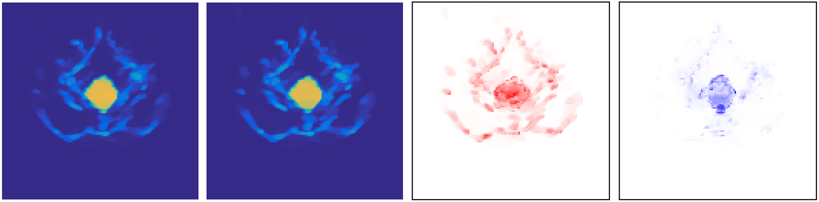
Potential for improving reconstruction from sub-sampled data demonstrated in various applications.

Osher, Burger, Goldfarb, Xu, Yin, 2006. *An iterative regularization method for total variation-based image restoration*, *Multiscale Modeling and Simulation*, 4(2):460-489.

Contrast Enhancement by Bregman Iterations



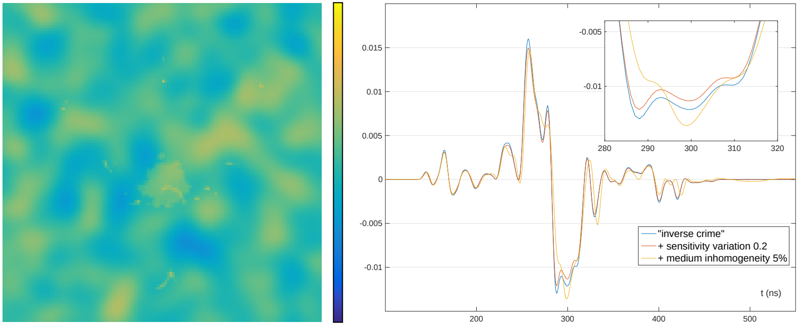
(a) TV+, full data (b) TV+Br, full data (c) $(p_{\mathrm{TV+Br}} - p_{\mathrm{TV+}})_+$, full data (d) $(p_{\mathrm{TV+Br}} - p_{\mathrm{TV+}})_-$, full data



(e) TV+, rSP-128 (f) TV+Br, rSP-128 (g) $(p_{\mathrm{TV+Br}} - p_{\mathrm{TV+}})_+$, rSP-128 (h) $(p_{\mathrm{TV+Br}} - p_{\mathrm{TV+}})_-$, rSP-128

sensor on top; inverse crime data sampled at Nyquist; max intensity proj., side view

!Data created by the same forward model used for reconstruction!



(a) $c_0 + \tilde{c}$ (b) (c) pressure-time courses

To avoid strong inverse crime:

- Generate data with perturbed, heterogeneous acoustic model.
- Model inhomogenous sensitivity and noise level of sensor channels.

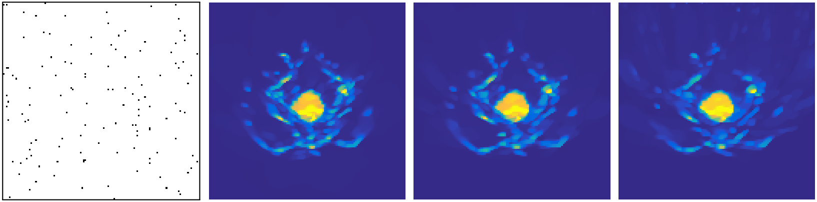
Nyquist Rates in Space and Time

- Up to now, "full data" corresponded to data sampled at Nyquist rates in space and time (numerical phantoms were band-limited in space).
- In experiments, the "full data" is usually already sub-sampled in space but over-sampled in time.
- Reconstruction on a finer spatial grid to exploit high frequency content of time series.

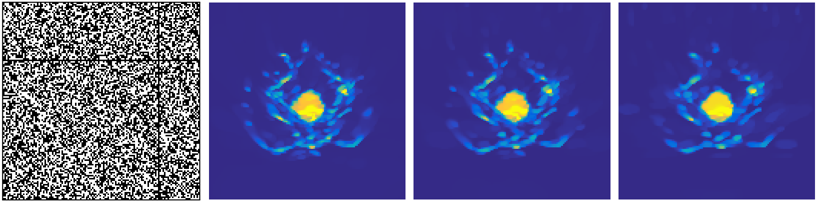
Example:

- Scan a 20mm×20mm with $\delta_x = 150\mu\text{m}$ (133×133 locations).
- Measured with temporal resolution of $\delta_t = 12\text{ns} \approx 83\text{MHz}$.
- Low-pass filtered to 20MHz.
- Reconstructing a signal limited to 20MHz with a sound speed of $1540\,\text{m}\,\text{s}^{-1}$ would required $\delta_x = 38.5\mu\text{m}$ and $\delta_t = 25\text{ns}$.

"Full data" is acquired on a grid which is 2 times too coarse (= factor 4).



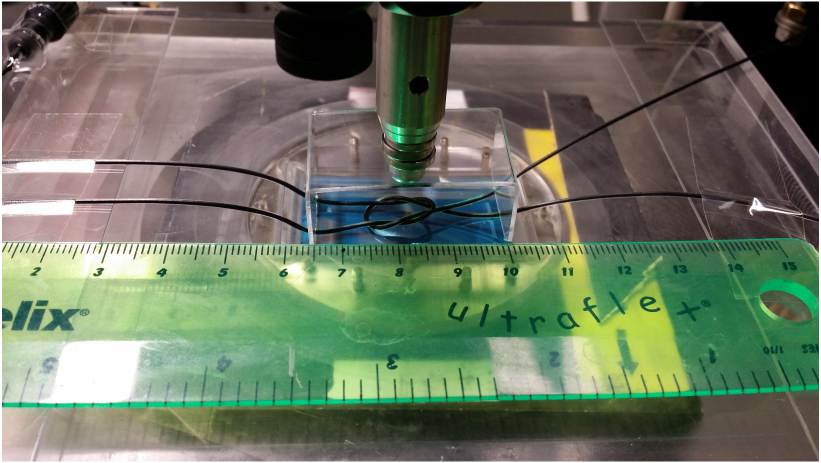
(d) single point (e) TV+Br, 1x (f) TV+Br, 8x (g) TV+Br, 32x



(h) patterned interrogation (i) TV+Br, 1x (j) TV+Br, 8x (k) TV+Br, 32x

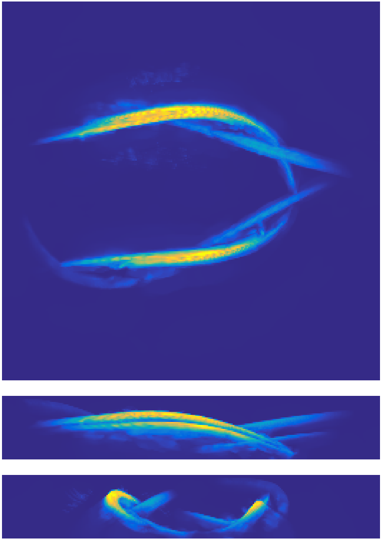
sensor on top; max intensity proj., side view

Leaving the Comfort Zone: Reproduction on Real Data

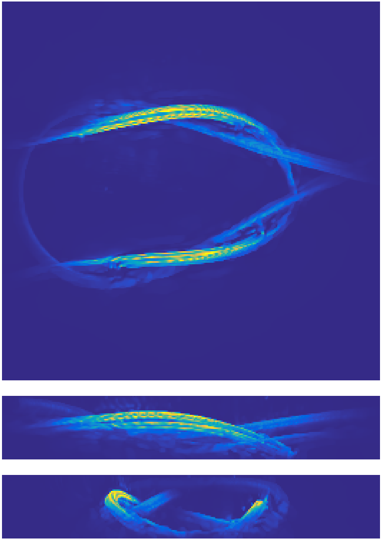


- Two polythene tubes filled with 10/100% ink.
- Stop-motion-style data acquisition of pulling one tube end.
- 45 frames (15min acquisition time per frame).
- Full data reconstructions to validate sub-sampling.

Full Data

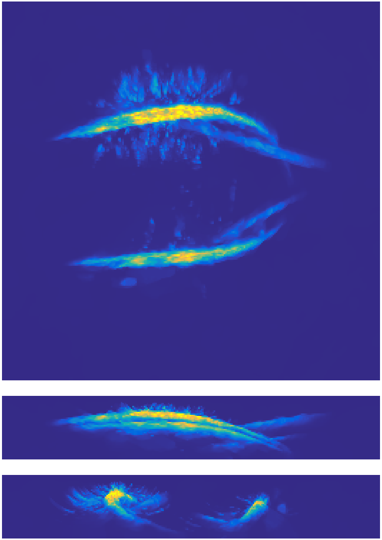


TR & TV denoising

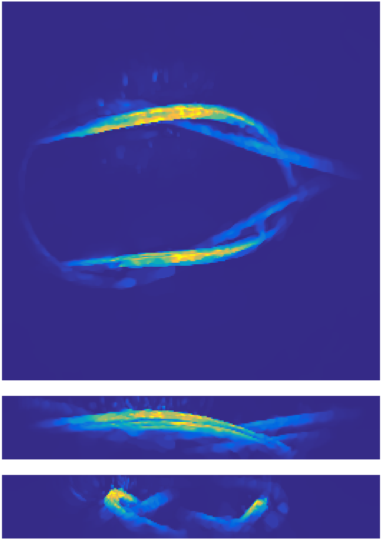


TV+

Random Point Sub-Sampled Data, 4x

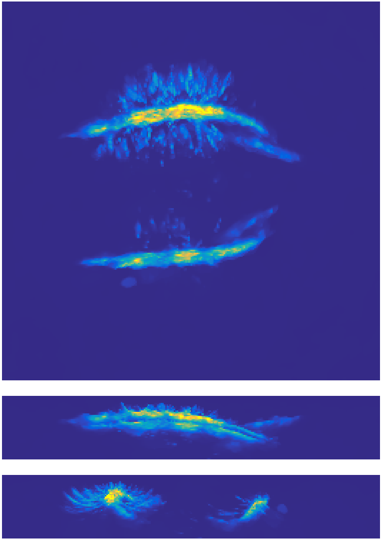


TR & TV denoising

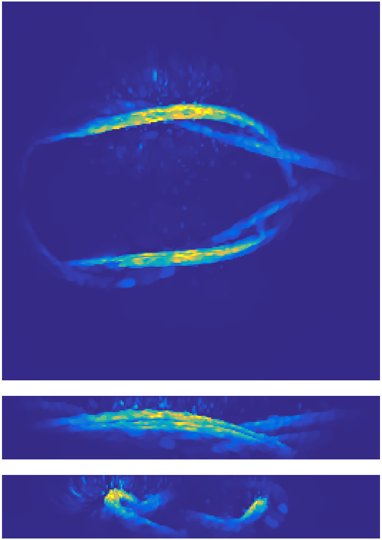


TV+

Random Point Sub-Sampled Data, 8x

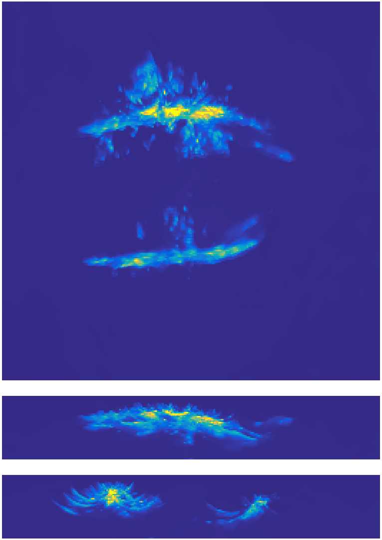


TR & TV denoising

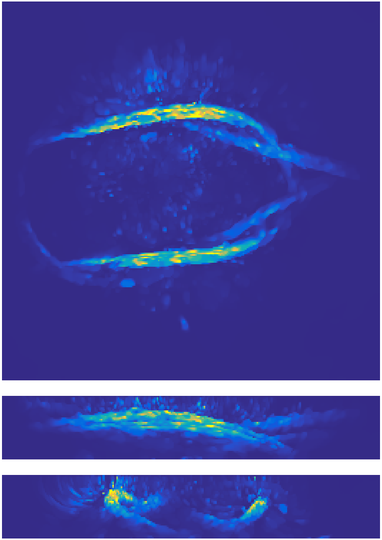


TV+

Random Point Sub-Sampled Data, 16x

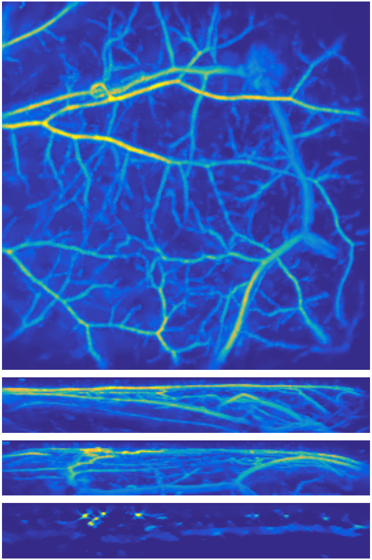


TR & TV denoising

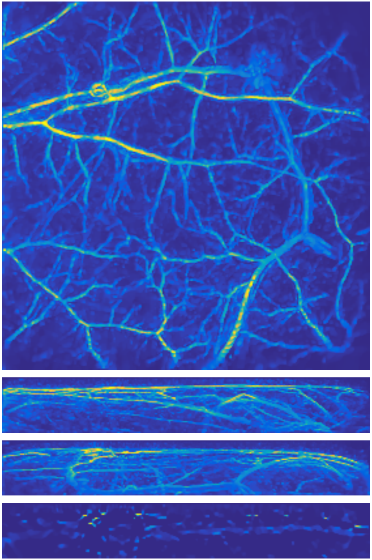


TV+

In Vivo Mesurements: Full Data

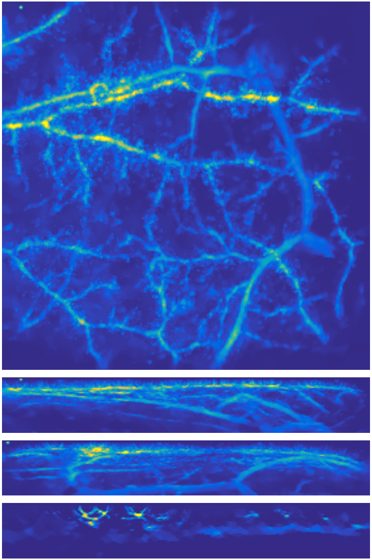


TR & TV denoising

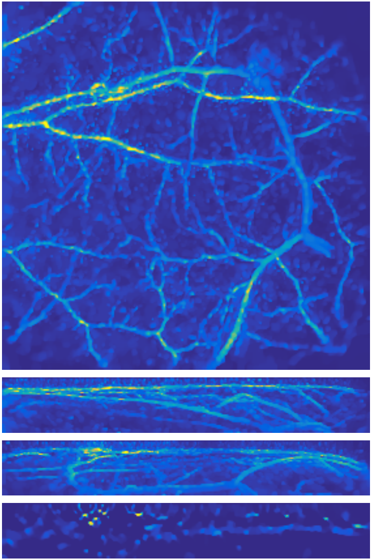


Bregman TV+

In Vivo Mesurements: 4x

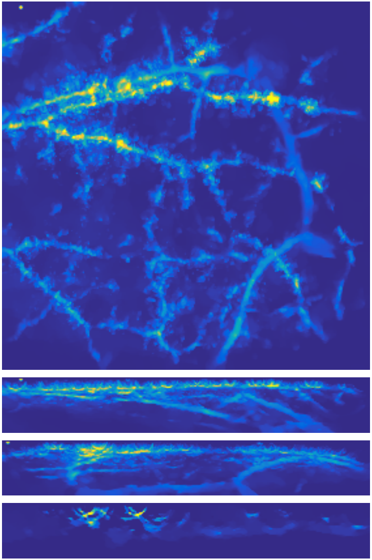


TR & TV denoising

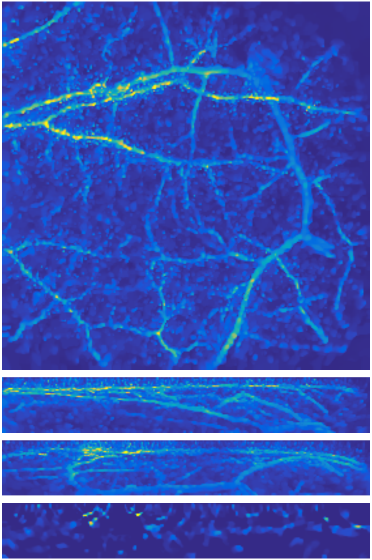


Bregman TV+

In Vivo Mesurements: 8x



TR & TV denoising



Bregman TV+

Reaching a high acceleration through sub-sampling requires:

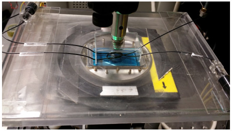
- Accurate model fit:
  - ! inhomogeneous optical excitation
  - ! uncertainty of acoustic parameters
  - ! inhomogeneity and defects of FP sensor
  - ! data artifacts by reflections / external sources

$\Longrightarrow$ Develop suitable, automatic pre-processing.
$\Longrightarrow$ Refine forward model used.

- Suitable regularization functionals:
  - ! TV is limited, especially for in-vivo data.
  - ! Experimental phantoms and in-vivo data are different.

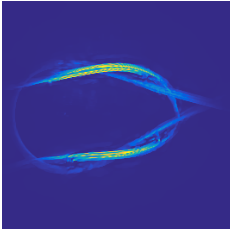
$\Longrightarrow$ Develop suitable regularizing functionals.

Spatio-Temporal Reconstruction



Continuous data acquisition

$\Longrightarrow$ tradeoff between spatial and temporal resolution.



Different dynamic models:

- Low-Rank (functional imaging with static anatomies/QPAT).
- Low-Rank + sparsity.
- Tracer uptake/wash-in models.
- Perfusion models.
- Needle guidance
- Optical flow constraints for joint image reconstruction and motion estimation.

Challenges of fast, high resolution 3D PA sensing:

- Nyquist requires several thousand detection points.
- Sequential schemes are slow.
- Parallelized schemes are prohibitively expensive.
- Crucial limitation for clinical, spectral and dynamical PAT.

Acceleration through sub-sampling:

- Exploit low spatio-temporal complexity of many targets.
- Acceleration by sub-sampling the incident wave field to maximize non-redundancy of data.
- Requires development of novel scanners.
- Demonstrated for Fabry-Pérot interferometer.

Summary II

Results:

- Novel sensing systems are developed.
- Standard reconstruction methods fail on sub-sampled data.
- Adjoint PAT operator allows to use variational approaches.
- Sparse variational regularization gives promising results for sub-sampled data.
- Demonstrated on simulated, experimental phantom and in-vivo data.

Challenges:

- Realizing this potential with experimental data requires model refinement/calibration and development of pre-processing.
- High computational complexity.

Outlook:

- Spatio-temporal variational models to exploit temporal redundancy.
- More suitable regularization functionals.

- **Arridge, Betcke, Cox, L, Treeby, 2015**. *On the Adjoint Operator in Photoacoustic Tomography*, (*submitted, arXiv:1602.02027*).
- **Arridge, Beard, Betcke, Cox, Huynh, L, Ogunlade, Zhang, 2016**. *Accelerated High-Resolution Photoacoustic Tomography via Compressed Sensing*, (*almost submitted*).

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NVIDIA

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UCL

Thank you for your attention!

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k-Wave

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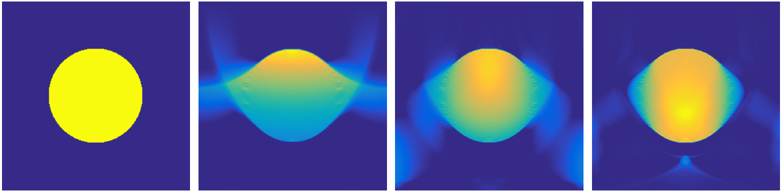
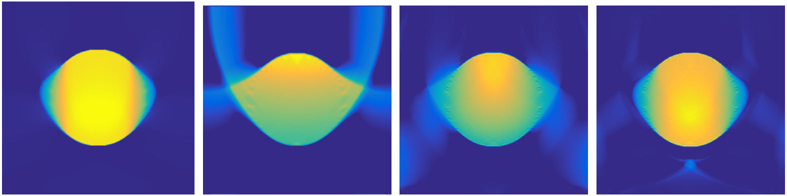
Iterative Schemes: Adjoint vs. Time Reversal

$$p^{k+1} = \Pi\left(p^k - \theta B\left(Ap^k - f\right)\right)$$



(a) Ground truth p_0 (b) TR (c) iTR (d) iTR+



(e) TV+ (f) BP (g) LS (h) LS+

sensor on top; 2D slices at $y = 128$ through the 3D reconstructions.