Centrum Wiskunde & Informatica

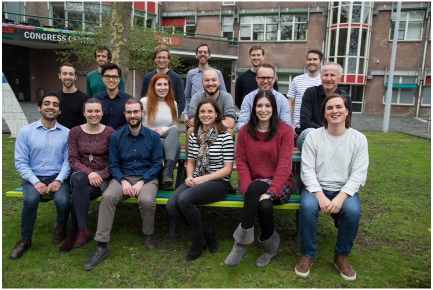
Challenges of Mathematical Image Reconstruction

Felix Lucka

DIAMANT symposium
Eindhoven
4 April 2019

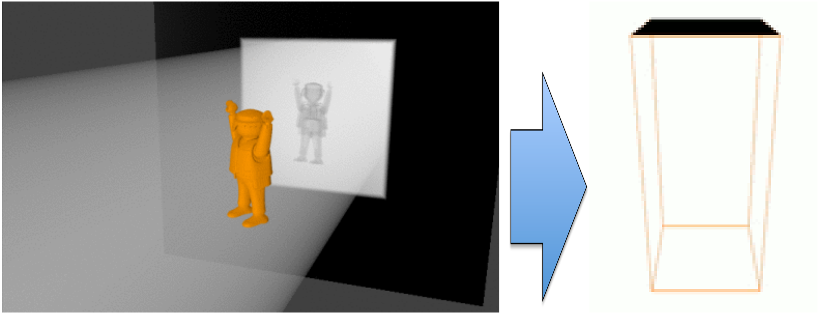
Introduction and Overview

Computational Imaging @ CWI



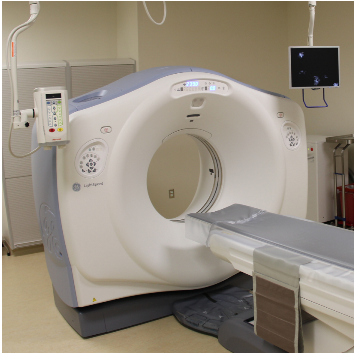
- headed by **Joost Batenburg**, 18 members
- mathematics, computer science & (medical) physics
- advanced computational techniques for 3D imaging
- (inter-)national collaborations from science, industry & medicine
- one of the two main developers of the **ASTRA Toolbox**
- **FleX-ray Lab**: custom-made, fully-automated **X-ray CT** scanner linked to large-scale computing hardware

X-ray Computed Tomography (CT)

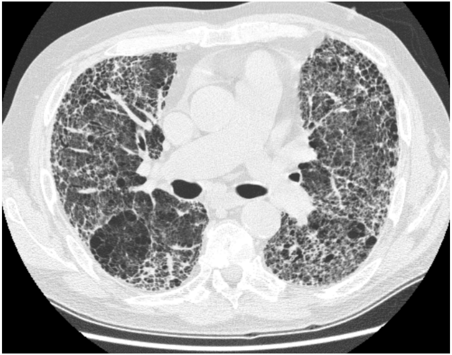


- X-rays (high-energy photons) get **attenuated** by matter
- 3D attenuation image **computed** from different 2D projections

X-ray Computed Tomography (CT)



(a) Modern CT scanner



(b) CT scan of a patient's lung

Source: Wikimedia Commons

Imaging Across Disciplines

Observational astronomy

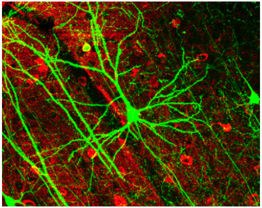
Life and material science
microscopy

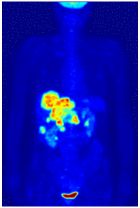
Medical imaging
CT, MRI, PET, SPECT, US...

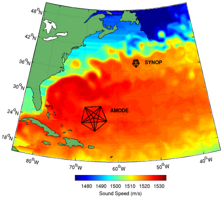
Geophysical imaging
(electrical) resistivity, seismic
(ground-penetrating) radar, ...

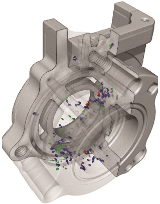
Remote sensing
earth science, military & intelligence

Industrial process imaging









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Imaging Across Disciplines

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Medical imaging

CT, MRI, PET, SPECT, US...

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(ground-penetrating) radar, ...

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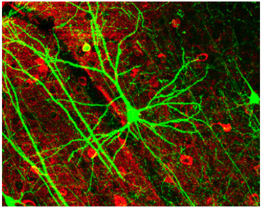
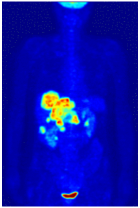
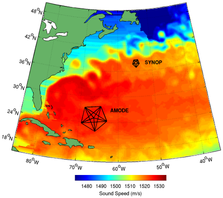
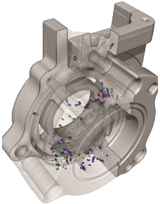








Source: Wikimedia Commons

Mathematical Imaging: *Reconstruct spatially distributed of quantities of interest from indirect observations through algorithms derived from rigorous mathematics.*

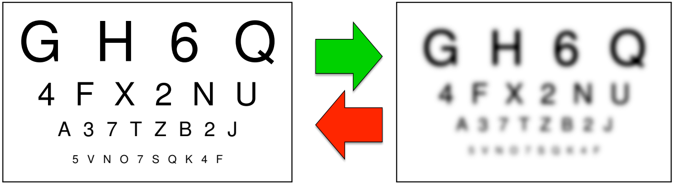
Imaging: An Inverse Problem

Inverse problem: Given **data** f recover **unknowns** u (image) from

$$\boxed{f = A(u) + \varepsilon}$$

- **Forward operator** A solution of **PDE** modelling underlying physics.

Imaging: An Inverse Problem



u $\qquad\qquad$ $A(u)$

Inverse problem: Given **data** f recover **unknowns** u (image) from

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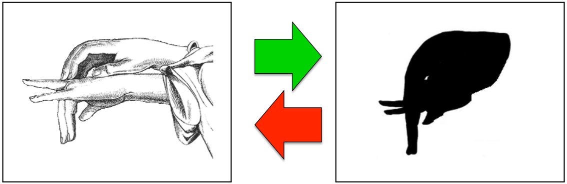
- Typical inverse problems are **ill-posed**.

Imaging: An Inverse Problem



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Inverse problem: Given **data** f recover **unknowns** u (image) from

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- **Forward operator** A solution of **PDE** modelling underlying physics.
- Typical inverse problems are **ill-posed**.
- Stable solution requires **a-priori information** on u.

Inverse Problems / Imaging Workflow

mathematical modeling
physics, PDEs, approximations

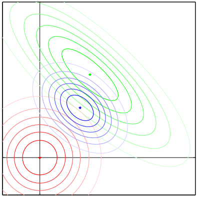
theoretical analysis
uniqueness, recovery conditions, stability

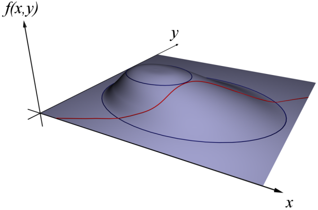
reconstruction/inference approach
regularization, statistical inference, machine learning

reconstruction algorithm
numerical linear algebra, PDEs, optimization, MCMC

large-scale computing
parallel computing, GPU computing

$$(s \cdot \nabla + \mu_a(x) + \mu_s(x))\, \phi(x, s) = q(x, s) + \mu_s(x) \int \Theta(s, s')\phi(x, s')ds'$$





Current Challenges in Computational Imaging

core development for new modalities:
hybrid imaging

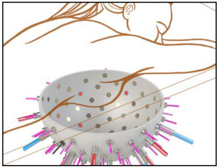
more from more:
multi-spectral, multi-modal, high resolution

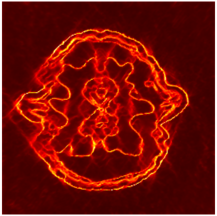
same from less:
low-dose, limited-view, compressed, dynamic

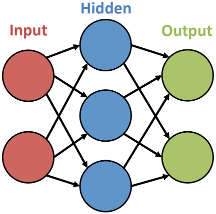
break the routine:
real-time, dose adaptation, zooming

uncertainty quantification & quantitative imaging

machine learning:
embedding, networks for 3D/4D, clinical training data







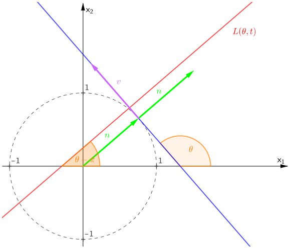
X-ray Computed Tomography

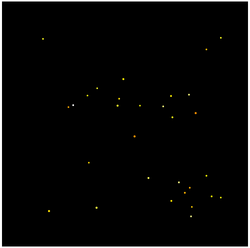
Mathematics of X-ray Computed Tomography (CT)

Beer-Lambert's law: Intensity of monochromatic ray passing through heterogeneous medium described by $\log\left(I_1/I_0\right) = -\int_l u(x)dx$.

$\rightarrow$ **integral geometry problem**, A reduces to **Radon transform**:

$$f(\theta, t) = \int_{L(\theta,t)} u(x)dx, \quad L(\theta, t) = \{x \in \mathbb{R} \,|\, x_1 \cos(\theta) + x_2 \sin(\theta) = t\}$$





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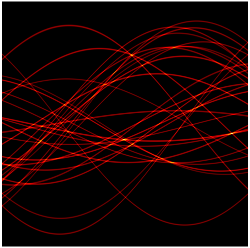

Au

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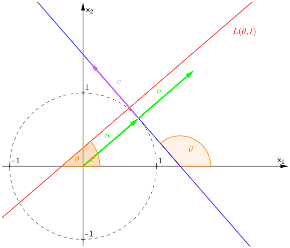

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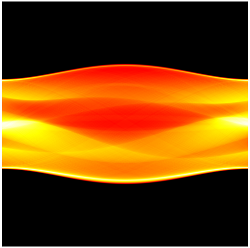

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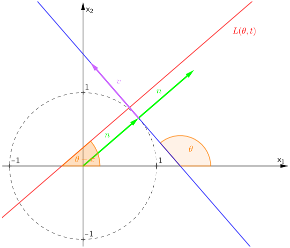

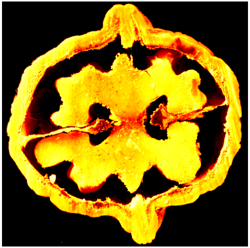

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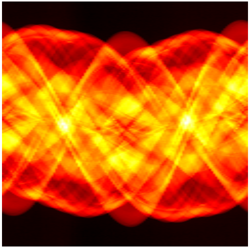


Au

Image Reconstruction Approaches

$$\boxed{f = Au + \varepsilon}$$

Analytical - determine A^{-1}, regularize it, discretize it

$$\hat{u} = A^{*}\mathcal{H}f \qquad\qquad (\textit{filtered backprojection} - \textit{FBP})$$

- ✓ efficient to implement and execute
- ! lack of flexibility for unconventional scanning set-ups
- ! severe artifacts for limited / sparse projection data
- ! hard to introduce a-priori knowledge

Image Reconstruction Approaches

$$\boxed{f = Au + \varepsilon}$$

Analytical - determine A^{-1}, regularize it, discretize it

Algebraic / variational - discretize and optimize via iterative scheme

$$\hat{u}_\lambda = \underset{u \in \mathcal{U}}{\operatorname{argmin}} \left\{ \frac{1}{2} \|Au - f\|_2^2 + \lambda \mathcal{J}(u) \right\}$$

- ! higher computational cost
- ✓ highly flexible, arbitrary geometries
- ✓ less artifacts for limited / sparse projection data
- ✓ introduction of a-priori knowledge possible

Image Reconstruction Approaches

$$\boxed{f = Au + \varepsilon}$$

Analytical - determine A^{-1}, regularize it, discretize it

Algebraic / variational - discretize and optimize via iterative scheme

Bayesian / statistical - explicit uncertainty modeling

$$p_{post}(u|f) = \frac{p_{like}(f|u)p_{prior}(u)}{p(f)}$$

! ! even higher computational cost

✓ rigorous assessment of solution's uncertainties

Image Reconstruction Approaches

$$\boxed{f = Au + \varepsilon}$$

Analytical - determine A^{-1}, regularize it, discretize it

Algebraic / variational - discretize and optimize via iterative scheme

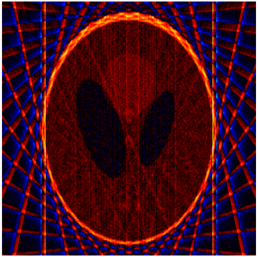
Bayesian / statistical - explicit uncertainty modeling

Deep learning - improve everything by trained DNNs

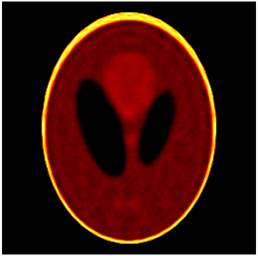
- ✓ extremely promising
- ✓ can be fast
- ! not well understood (yet)
- ! training data

Illustration of Different Reconstruction Methods

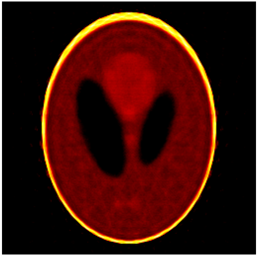
(a) true image



(b) FBP



(c) ART

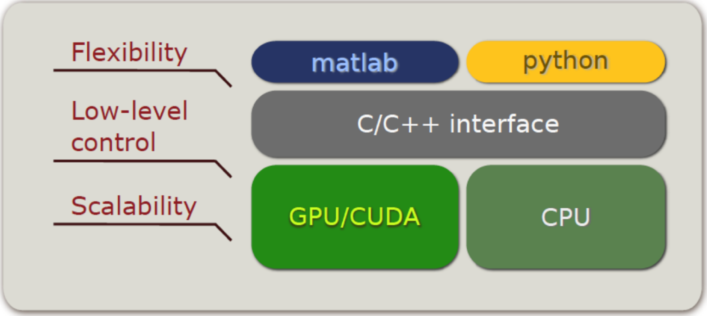


(d) SIRT

(e) TV regularization

ASTRA Toolbox

- open source software, developed by CWI and Univ. Antwerp
- provides scalable, high-performance GPU primitives for tomography
- flexible with respect to projection geometry
- featured in the NVIDIA CLARA Platform



www.astra-toolbox.com

Deep Learning with Convolutional Neuronal Networks

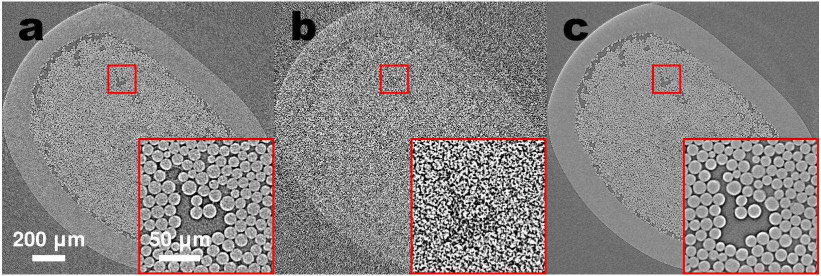
Approximate function $v = G(u)$ by **neuronal network** G_θ:

- G_θ: composition of **many computational units (layers)**
- layers: $y = \sigma\left(Wx + b\right)$
- W is convolution: **convolutional neuronal network (CNN)**
- θ: all free parameters
- **learning**: from **training set** $\{(u_i, v_i)\}_{i=1}^m$

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmin}} \left\{ \sum_i^m Loss\left(G_\theta(u_i), v_i\right) + \lambda \mathcal{J}(\theta) \right\}$$

- (stochastic) gradients via **backpropagation & automatic differentiation**

DNN for Removal of FBP Artefacts



2560x2560 tomography images of fiber composite.

Left: 1024 projections, *middle/right*: 128 projections

Pelt, Batenburg, Sethian, 2018. Improving Tomographic Reconstruction from Limited Data Using Mixed-Scale Dense Convolutional Neural Networks, *Journal of Imaging 4 (11), 128.*

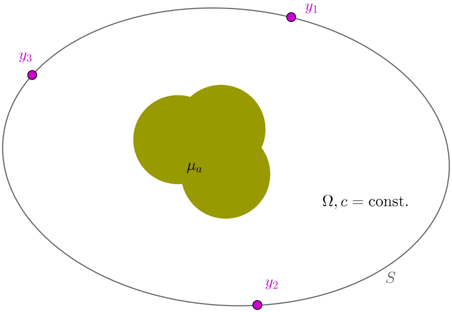
Pelt, Sethian, 2018. Mixed-scale dense network for image analysis, *PNAS 115 (2) 254-259.*

Photoacoustic and Ultrasound Tomography

Photoacoustic Imaging: Physical Principles

Optical Part

optical absorption coefficient: μ_a



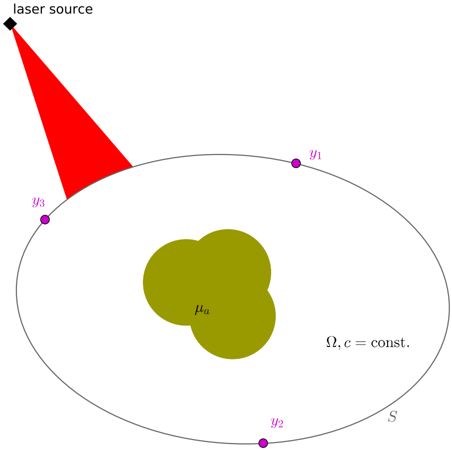
Acoustic Part

Photoacoustic Tomography: Physical Principles

Optical Part

optical absorption coefficient: μ_a

pulsed laser excitation: Φ



Acoustic Part

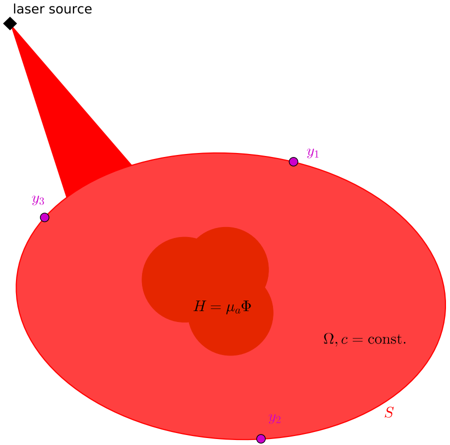
Photoacoustic Tomography: Physical Principles

Optical Part

optical absorption coefficient: μ_a

pulsed laser excitation: Φ

thermalization by chromophores: $H = \mu_a \Phi$



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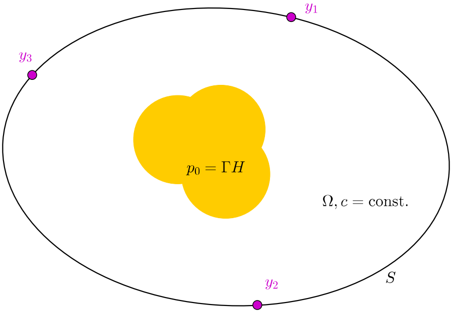
Acoustic Part

local pressure increase: $p_0 = \Gamma H$



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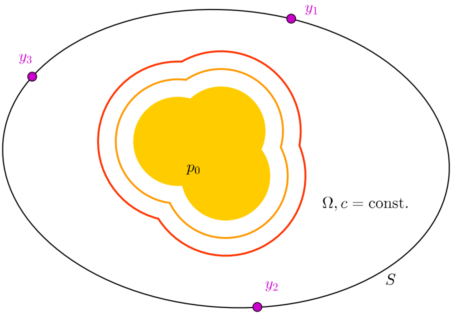
Acoustic Part

local pressure increase: $p_0 = \Gamma H$

elastic wave propagation: $p(x, t)$



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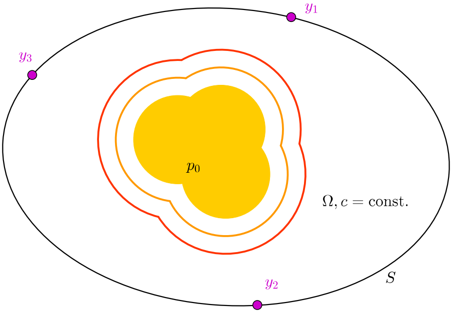
elastic wave propagation: $p(x, t)$

measurement of pressure time courses:

$$f_i(t) = p(y_i, t)$$



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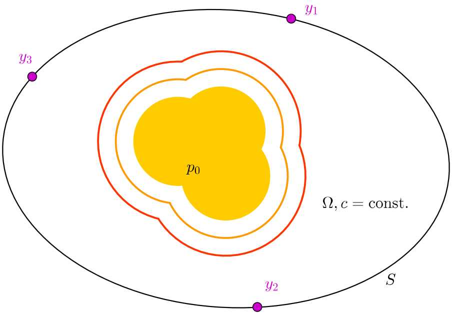


Photoacoustic effect

- **coupling** of optical and acoustic modalities.
- **"hybrid imaging"**
- **high optical contrast** sensed by **high-resolution** ultrasound.

Photoacoustic Tomography: Mathematical Formulation

(stationary) radiative transport equation (RTE)

$$(s \cdot \nabla + \mu_a(x) + \mu_s(x))\,\phi(x,s) = q(x,s) + \mu_s(x) \int \Theta(s,s')\phi(x,s')ds',$$

coupled with **acoustic wave equation**

$$p(x,t=0) = p_0 := \Gamma(x)\mu_a(x) \int \phi(x,s)ds, \qquad \partial_t p(x,t=0) = 0$$

$$(c(x)^{-2}\partial_t^2 - \Delta)p(x,t) = 0, \qquad f = Sp|_{\mathrm{M}\times[0,T]}$$

Hybrid inverse problem:

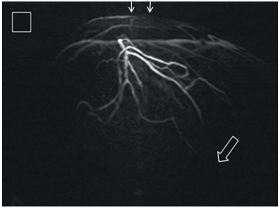
✓ acoustic initial value problem with boundary data

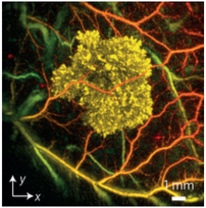
✓ optical parameter identification problem with internal data

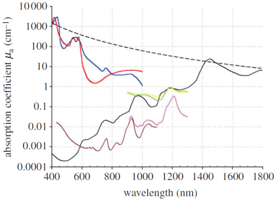
vs. *diffuse optical tomography (DOT):*

! optical parameter identification problem with boundary data

Photoacoustic Tomography: Applications

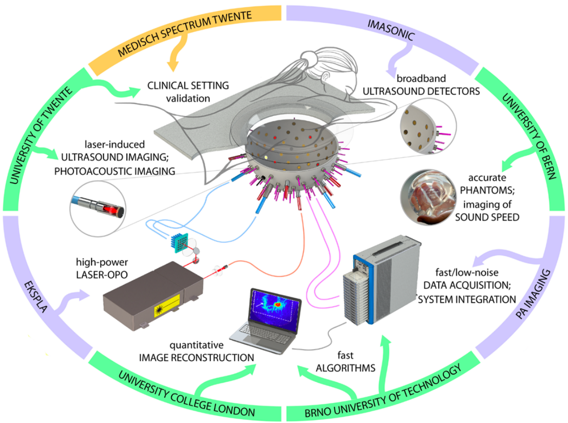






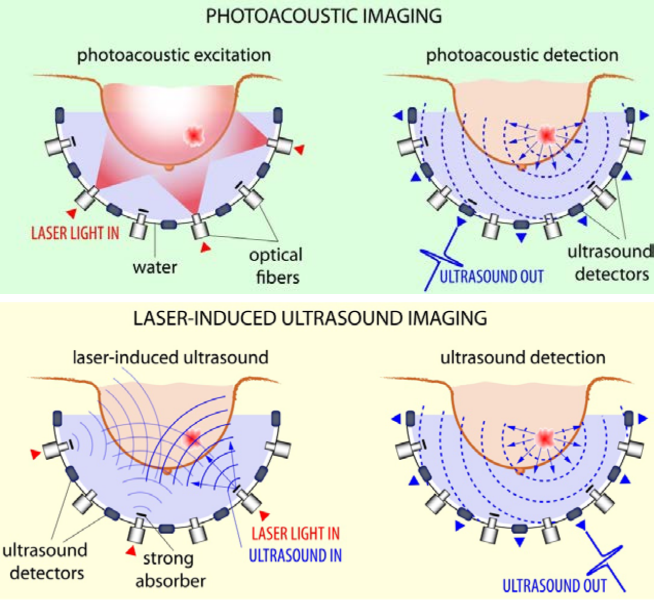
- High contrast for light-absorbing structures in soft tissue.
- Gap between oxygenated and deoxygenated blood.
- Different wavelengths allow **quantitative spectroscopic examinations**.
- Use of contrast agents for **molecular imaging**.
- **Extremely promising future imaging technique!**

H2020 Project: Photoacoustic Mammography Scanner



- **Real-time** photoacoustic imaging
- **Multi-modal**: joint **ultrasound CT (USCT)** and PA imaging.
- **Multi-spectal**: quantitative **sO_2 imaging**.

H2020 Project: Photoacoustic Mammography Scanner



3D Wave Propagation Methods for PAT and USCT

k-space pseudospectral time domain method:

B. Treeby and B. Cox, 2010. k-Wave: MATLAB toolbox for the simulation and reconstruction of photoacoustic wave fields, *Journal of Biomedical Optics.*

derivation and discretization of **adjoint PAT operator** A^*:

Arridge, Betcke, Cox, L, Treeby, 2016. On the Adjoint Operator in Photoacoustic Tomography, *Inverse Problems 32(11).*

approximation via deep learning:

Hauptmann, Cox, L, Huynh, Betcke, Beard, Arridge, 2018. Approximate k-space models and Deep Learning for fast photoacoustic reconstruction, *MLMIR 2018.*

Radiative Transport Equation in 3D

$$(s \cdot \nabla + \mu_a + \mu_s)\,\phi(x,s) = q + \mu_s \int \Theta(s,s')\phi(x,s')ds', \quad \Phi(x) = \int \phi(x,s)ds$$

! $(x,s) \in \mathbb{R}^5 \rightsquigarrow$ direct FEM infeasible.

Diffusion approximation:

$$(\mu_a - \nabla \cdot \kappa(x)\nabla)\,\Phi(x) = \int q(x,s)ds, \quad \kappa = \frac{1}{3(\mu_a + \mu_s(1-g))}$$

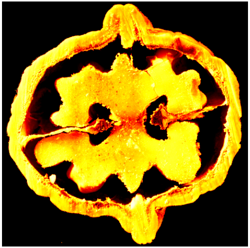
Schweiger, Arridge, 2014. The Toast++ software suite for forward and inverse modeling in optical tomography, *Journal of Biomedical Optics.*

Alternative: GPU-based **Monte Carlo** estimate of transport density

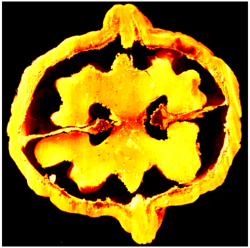
CWI

Compressed Sensing and Dynamic Imaging

Sparsity & Compressed Sensing



(a) 100%

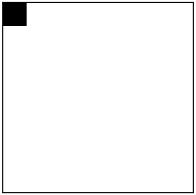


(b) 10%

(c) 1%

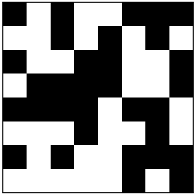
- **sparsity** traditionally used for compression of **Nyquist data**.
- Nyquist sampling: too much time/radiation!
- directly sense non-redundant information? $\rightarrow$ **compressed sensing**

Accelerated Imaging via Compressed Sensing



Beat Nyquist for objects with **low spatio-temporal complexity** by **incoherent sub-sampling**,

$$f^c = Cf = C(Au + \varepsilon)$$



combined with **sparsity-constrained variational image reconstruction**:

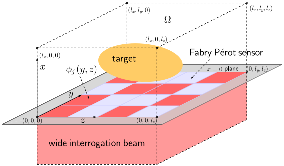
$$\hat{u}_\lambda = \operatorname*{argmin}_{u \in \mathcal{U}} \left\{ \frac{1}{2} \| CAu - f \|_2^2 + \lambda \mathcal{J}(u) \right\}$$

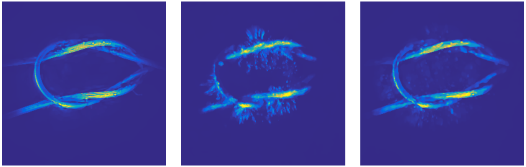
! Development of novel acquisition systems.

! Iterative, first-order methods for non-smooth optimization.

! Matrix-free implementation of A, A^*.

Accelerated 3D PAT via Compressed Sensing





- ✓ development of compressed sensing PAT scanners
- ✓ implementation of **sparse regularization** schemes
- ✓ realistic simulated, experimental and *in-vivo* data
- ✓ **significant acceleration** with minor loss of quality
- ✓ further improvement through **deep learning**

Arridge, Beard, Betcke, Cox, Huynh, L, Ogunlade, Zhang, 2016. Accelerated High-Resolution Photoacoustic Tomography via Compressed Sensing, *PMB.*

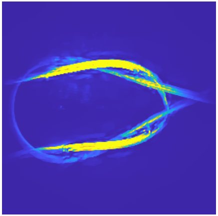
Hauptmann, L, Betcke, Huynh, Adler, Cox, Beard, Ourselin, Arridge, 2018. Model-Based Learning for Accelerated, Limited-View 3-D Photoacoustic Tomography, *IEEE-TMI.*

Spatio-Temporal Reconstruction: 4D PAT

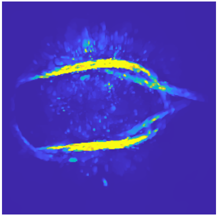
Dynamic compressed sensing:

$$f_t^c = C_t f_t = C_t(Au_t + \varepsilon_t)$$

Limitations of frame-by-frame $\longrightarrow$



full data



16x acc. (6.25%)

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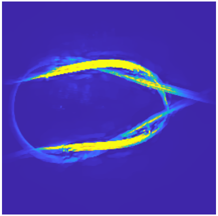

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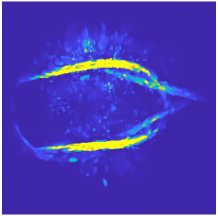

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Spatio-temporal image reconstruction:

Parametric models (shift, stretch, etc.): simple and nice if applicable.

Non-parametric models, e.g., spatio-temporal variational schemes:

$$\hat{u} = \operatorname*{argmin}_{u \in \mathcal{U}} \left\{ \sum_t^T \frac{1}{2} \left\| C_t A\, u_t - f_t^c \right\|_2^2 + \lambda \mathcal{R}(u) \right\}$$

- space-time decomposition (structured low-rank)
- more sophisticated: joint reconstruction of image and dynamics.

Spatio-Temporal Reconstruction: 4D PAT

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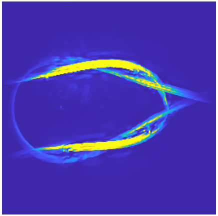

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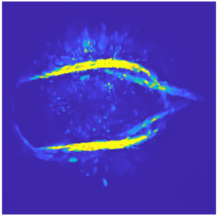


16x acc. (6.25%)

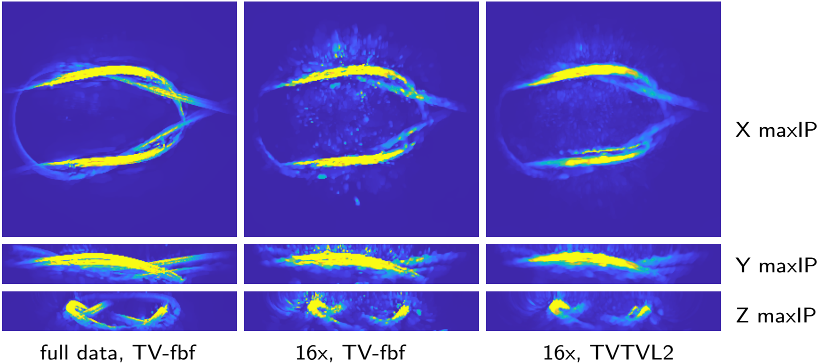
$$(\hat{u}, \hat{v}) = \underset{u \in \mathcal{U}, v \in \mathcal{V}}{\operatorname{argmin}} \left\{ \sum_t^T \frac{1}{2} \| C_t A\, u_t - f_t^c \|_2^2 + \alpha \mathcal{J}(u_t) + \beta \mathcal{H}(v_t) + \gamma \mathcal{S}(u, v) \right\}$$

$\mathcal{S}(u, v)$ enforces PDE model of dynamics, e.g., optical flow equation:

$$\partial_t u(x,t) + (\nabla_x u(x,t))\, v(x,t) = 0$$

Burger, Dirks, Schönlieb, 2018. A Variational Model for Joint Motion Estimation and Image Reconstruction, .

Dynamic Compressed Sensing with Optical Flow Constraints



✓ Proof-of-concept for 4D CS PAT data.

! High dimensional, non-smooth, bi-convex optimization problem.

L, Huynh, Betcke, Zhang, Beard, Cox, Arridge, 2018. Enhancing Compressed Sensing 4D Photoacoustic Tomography by Simultaneous Motion Estimation, *SIAM Journal on Imaging Sciences 11:4, 2224-2253.*

Dynamic Compressed Sensing with Deep Learning

Hauptmann, Arridge, L, Muthurangu, Steeden, 2018. Realtime cardiovascular MR with spatiotemporal artifact suppression using deep learning - proof of concept in congenital heart disease, *Magnetic Resonance in Medicine.*

Summary

Summary

- imaging has broad range of applications
- mathematically: **inverse problem** of reconstructing distributed quantities from indirect observations
- stable solution requires **a-priori information**
- **mathematical modeling**, (solving) **PDEs**, **numerical optimization**
- 3D: high performance computing
- **compressed sensing** and **dynamic/spectral** imaging
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Thank you for your attention!

Arridge, Betcke, Cox, L, Treeby, 2016. On the Adjoint Operator in Photoacoustic Tomography, *Inverse Problems 32(11).*

Arridge, Beard, Betcke, Cox, Huynh, L, Ogunlade, Zhang, 2016. Accelerated High-Resolution Photoacoustic Tomography via Compressed Sensing, *PMB 61(24).*

Hauptmann, L, Betcke, Huynh, Adler, Cox, Beard, Ourselin, Arridge, 2018. Model-Based Learning for Accelerated, Limited-View 3-D Photoacoustic Tomography, *IEEE-TMI 37(6).*

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